Quantum Fermi Liquid Decription of 1D Electronic Systems and Topological Spin-Charge Separation Effect

Alla Dovlatova (a), Dmitri Yerchuck (b), Felix Borovik (b)
(a) - M.V.Lomonosov Moscow State University, Moscow, 119899,
(b) Heat-Mass Transfer Institute of National Academy of Sciences of RB, Brovka Str.15, Minsk, 220072
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It is shown, that spin-charge separation effect can be realized in 1D quantum Fermi-liquids. It has topological soliton origin in distinction from well known spinon-holon spin-charge separation effect in Luttinger liquids and electronic systems like them. It is the result of the generalization of Su-Schrieffer-Heeger (SSH) model of organic 1D conductors in the frames of 1D quantum Fermi liquid description. In its turn it allows to extend the limits of the applicability of SSH-model to the systems with strong electron-phonon interaction and (or) strong electron-photon interaction.

TOC Graphic

$$E_k^{(c)}(u) = \frac{\mathcal{Q}^2\Delta_k^2 - \epsilon_k^2}{\sqrt{\epsilon_k^2 + \mathcal{Q}^2\Delta_k^2}}, \qquad E_k^{(v)}(u) = \frac{\epsilon_k^2 - \mathcal{Q}^2\Delta_k^2}{\sqrt{\epsilon_k^2 + \mathcal{Q}^2\Delta_k^2}} \tag{1}$$

$\mathcal{Q} = 1 \rightarrow$ Energy of quasiparticles in 1D quantum Fermi gas,
$\mathcal{Q}$ determined by

$$[1 + \frac{\alpha_2}{2\alpha_1}\sum_k\sum_s \frac{\mathcal{Q}\Delta_k \sin ka}{\sqrt{\epsilon_k^2 + \mathcal{Q}^2\Delta_k^2}}(n_{k,s}^{(c)} - n_{k,s}^{(v)})] = \mathcal{Q}, \tag{2}$$

$\rightarrow$ Energy of quasiparticles in 1D quantum Fermi liquid

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I. INTRODUCTION

Spin-charge separation effect in 1D-systems is associated in the literature usually with Luttinger liquid (LL) behavior of electronic system of 1D conductors, which cannot be described in the frames of Landau Fermi liquid (LFL) concept. The most known example of 1D-system with LL behavior is doped Mott-Hubbard insulator in the metallic regime, for which the idea of spin-charge separation was originally introduced by Anderson in 1987 [1], [2], [3]. Spin-charge separation in Anderson approach means the existence of two independent elementary excitations, charge-neutral spinon and spinless holon, which carry spin 1/2 and charge $+e$ respectively. Similar spin-charge separation effect may be mathematically realized in the so-called slave-particle representation [4] of the $t - J$ model.

It has to be remarked, that both the models LL and LFL are the models of ideal quantum liquids, which do not take into account the nonlinearity of the fermion spectrum on the one hand and electron-phonon interactions on the other hand. Let us remember, that the definition of the notion "Luttinger liquid" is based on the simplification, determined by linearization of the generic spectrum of particles in neighborhood of Fermi points in k-space. Given simplification has led to divergencies, arising in the perturbation theory in application to LFL-model for 1D systems. In given case, a different paradigm has developed from Tomonaga idea [5], that the low-energy degrees of freedom of a 1D Fermi gas are completely collective, and the development of the "bosonization" technique. The conceptual starting point for the bosonization of the Fermi surface is the Luttinger theorem [6], from here the name "Luttinger liquid" arose. However, how it was remarked in [7], even for a linear spectrum, the bosonic or fermionic languages may be used equally comfortably and both offer their particular benefits. The advantage of the former is the direct relation between the bosonic modes and the density response functions. On the other hand, the fermionic description connects to the well-known physics of the Fermi edge problem. Moreover they have shown, that in order to calculate the dynamic response functions in the case of the nonlinearity of the fermion spectrum, it is convenient to translate the bosonic spin and charge modes into fermionic quasiparticles, spinons and holons. For a nonlinear spectrum, the fermionic basis is superior because it avoids divergencies arising in the bosonic perturbation theory [7].

At the same time the idea of spin-charge separation was explicitly treated for the first time already in 1974 by Luther and Emery [8] in the context of a continuum limit of the $1D$ electron gas theory. They have shown, that the Hamiltonian $\hat{H}_{1DEG}$ of the 1D electron gas can

be represented in the form of

$$\hat{H}_{1DEG} = \hat{H}_c[\phi_c] + \hat{H}_s[\phi_s] + \hat{H}_{irr}[\phi_c, \phi_s], \qquad (3)$$

where $\hat{H}_c[\phi_c]$ and $\hat{H}_s[\phi_s]$ are, respectively, the Hamiltonians, which govern the dynamics of the spin and charge fields, ϕ_c and ϕ_s, respectively, and $\hat{H}_{irr}[\phi_c, \phi_s]$ consists of terms that can be neglected in the long wave-length limit.

The related model, which describes spin-charge separation is the model of the formation of solitons with fractional fermion number. General idea belongs to Jackiw and Rebbi [10]. They pay attention to the field theories, especially in one spatial dimension, which lead solitons' formation with fractional fermion number. However the concrete realization of given idea in condensed matter physics belongs to Su, Schrieffer, and Heeger. The model, proposed by Su, Schrieffer, and Heeger (SSH-model) with spin-charge separation to be the basis phenomenon is the model of conjugated organic 1D-conductors.

Specifically, what Su, Schrieffer, and Heeger showed, is that when an electron is added to an neutral trans-polyacetylene (t-PA) chain, it can break up into two pieces, one of which carries the electron's charge and the other its spin. The real significance of the SSH-soliton model of t-PA is that it introduced a new paradigm into the field. Triuph of the SSH model is not occasional. The formulation of the model is very simple from mathematical viewpoint and the simplicity itself is the great advantage of the model. At the same time it demonstrates the deep physical insight of Su, Schrieffer and Heeger in the field, which was argued in [9], [14], [22]. Really, although the term, which takes into consideration the electron-electron correlations is not presented in SSH Hamiltonian in explicit form, it, in fact, is represented in implicit form. Really, the electron-electron interaction is easily can be taken into account in the model by means of its renormalization into electron-phonon interaction with effective coupling parameter [9], [15], [16]. It is very interesting, that the very similar theoretical result on the possibility to renormalize electron-phonon coupling into equivalent electron-electron interactions was obtained independently many years after in 2006 in [20]. It was shown, that, the spin-1/2 Holstein model could be mapped onto the negative-U Hubbard model with an effective dynamical attraction $U_{eff}(\omega)$, dependence of which on the frequency ω is given by the relation $U_{eff}(\omega) = \lambda/(1 - \omega^2/\omega_0^2)$, where λ is the electron-phonon coupling constant in energetic units, ω_0 is the bare phonon frequency. The merit of SSH-model, consisting in the choose of the only dimerization coordinate u_n of the n-th CH-group, $n = \overline{1, N}$, along chain molecular-symmetry axis x to be substantial for determination of main physical properties of the material and neglecting by the other five degrees of freedom, that is the degrees of freedom, which are relevant to the bonds with the directions not coinciding with chain molecular-symmetry axis direction, was commented in [14] and in [22]. Given possibility is the consequence of general principle, which was proposed by Slater at the earliest stage of quantum physics era already in 1924 [17]. However the most merit of SSH-model, which demonstrates a very deep insight of authors in the field, is without any comment up to now. In fact, the only given model in the condensed matter physics of dynamic electronic systems takes into consideration in explicit form the generic coupling between operators of creation and annihilation of two quantum fields - between the operators of the field corresponding to electron subsystem and the operators of the field of lattice deformation system. The simplest example, when generic coupling beween given two fields is taking into account, is quantum chemistry calculations of the structure of point centers in crystals. It is well known, that by the change of the charge state of any point center in crystal lattice, the atomic relaxation of neighbourhood host lattice atoms has to be taken into account. In dynamical case it corresponds to phonon absorption or generation. It seems to be evident, that in SSH-model the opreators of phonon subsystem are represented through operators of elecronic subsystem taking into account given coupling in explicit form. Let us remark, that usually given operators are considered independently on each other, which can lead to distortion of description of real physical processes.

It seems to be interesting, that there are qualitative differences by description of spin-charge separation effects in 1D systems between SSH mechanism and Anderson mechanism, which is applicable, in particular, to the 1D models, in which 1D correlated electronic systems are properly described by the Luttinger liquid theory. The main differerence consists in the role of phonon effects in spin-charge separation phenomenon. For instance, the role of phonon effects on spin-charge separation in one dimension by Anderson mechanism was studied in [20], through the calculation of one-electron spectral functions in terms of the cluster perturbation theory together with an optimized phonon approach. The 1D Holstein-Hubbard model has been used. It was found that the retardation effect, which is the consequence of the finiteness of phonon frequency suppresses the spin-charge separation and eventually makes it invisible in the spectral function. At the same time electron-phonon interaction plays the essential role for spin-charge separation presence in SSH-model. Let us remark, that there is in existing variant of SSH-model an upper limit on the value of electron-phonon coupling constant. It is consequence of the treatment of electron-phonon coupling to be the linear term in expansion of the only hopping integral of tight-binding model about the undimerized state. Given restriction was discussed in [21] and the maximum for allowed value of electron-phonon coupling constant α was evaluated to be ≈ 1.27. We will show, that given restriction can be remitted in developed variant of SSH-model.

Su, Schrieffer, Heeger [11], [12] describe mathematically the chain of t-PA by considering it to be Fermi gas in the sense, that the electron-electron interaction is

not taken into considereton in explicit form, although electron-phonon interaction is taken into account. We see, therefore, that SSH-model takes intermediate place between Fermi gas and Fermi liquid quantum models. The main idea of given work is the development of SSH-model in the frames of completely 1D Fermi liquid description, that determines the aim - to show the possibility to describe the 1D electronic systems in the frames of quantum Fermi liquids, first of all to establish the possibility of the phenomenon of spin charge separation in 1D quantum Fermi liquids.

II. RESULTS AND DISCUSSION

We will start from Hamiltonian

$$\hat{\mathcal{H}}(u) = \hat{\mathcal{H}}_0(u) + \hat{\mathcal{H}}_{\pi,t}(u) + \hat{\mathcal{H}}_{\pi,u}(u). \quad (4)$$

Like to works [11], [12] we will consider Born-Oppenheimer approximation. The first term in (4) is the following

$$\hat{\mathcal{H}}_0(u) = \sum_m \sum_s (\frac{\hat{P}_m^2}{2M^*}\hat{a}^+_{m,s}\hat{a}_{m,s} + Ku_m^2\hat{a}^+_{m,s}\hat{a}_{m,s}). \quad (5)$$

It represents itself the sum of operator of kinetic energy of CH-group motion (the first term in (5)) and the operator of the σ-bonding energy (the second term). Coefficient K in (5) is effective σ-bonds spring constant, M^* is total mass of CH-group, u_m is configuration coordinate for m-th CH-group, which corresponds to translation of m-th CH-group along the symmetry axis z of the chain, $m = \overline{1,N}$, N is number of CH-groups in the chain, $\hat{P}_m$ is operator of impulse, conjugated to configuration coordinate u_m, $m = \overline{1,N}$, $\hat{a}^+_{m,s}$, $\hat{a}_{m,s}$ are creation and annihilation operators of creation or annihilation of quasipartile with spin projection s on the m-th chain site in σ-subsystem of t-PA.

The second term in (4) can be represented in the form of two components, and it is

$$\hat{\mathcal{H}}_{\pi,t}(u) = \hat{\mathcal{H}}_{\pi,t_0}(u) + \hat{\mathcal{H}}_{\pi,t_1}(u), \quad (6)$$

where

$$\hat{\mathcal{H}}_{\pi,t_0}(u) = \sum_m \sum_s [t_0(\hat{c}^+_{m+1,s}\hat{c}_{m,s} + \hat{c}^+_{m,s}\hat{c}_{m+1,s})]. \quad (7)$$

It is the resonance interaction (hopping interaction in tight-binding model approximation) of quasiparticles in π-subsystem of t-PA electronic system, which is considered to be Fermi liquid, and in which the only constant term in Taylor series expansion of resonance integral about the dimerized state is taking into account. Here $\hat{c}^+_{m,s}$, $\hat{c}_{m,s}$ are creation and annihilation operators of creation or annihilation of quasipartile with spin projection s on the m-th chain site in π-subsystem of t-PA. The second term in (6) is

$$\hat{\mathcal{H}}_{\pi,t_1}(u) = \sum_m \sum_s [t_1\hat{c}^+_{m,s}\hat{c}^+_{m+1,s}\hat{c}_{m+1,s}\hat{c}_{m,s}]. \quad (8)$$

It represents itself the term, which is proportional to the constant first term in Taylor series expansion about the dimerized state of potential energy of the pairwise interaction of quasiparticles in π-subsystem between themselves. The third term in (4) is

$$\hat{\mathcal{H}}_{\pi,u}(u) = \sum_m \sum_s [(-1)^m 2\alpha_1 u(\hat{c}^+_{m+1,s}\hat{c}_{m,s} + \hat{c}^+_{m,s}\hat{c}_{m+1,s}) + (-1)^m 2\alpha_2 u\hat{c}^+_{m,s}\hat{c}^+_{m+1,s}\hat{c}_{m+1,s}\hat{c}_{m,s}]. \quad (9)$$

It consists, correspondingly, of the terms, which are proportional to linear terms in Taylor series expansion about the dimerized state of the resonance interaction of quasiparticles in π-subsystem of t-PA electronic system and potential energy of the pairwise interaction of quasiparticles in π-subsystem between themselves. There is taken into account, that in Born-Oppenheimer approximation in perfectly dimerized chain the coordinates $\{u_m\}$, $m = \overline{1,N}$, can be represented in the form $\{u_m\} = \{(-1)^m u\}$, where u is displacement amplitude, corresponding to minimum of ground state energy to be a function of displacement value. It seems to be evident, taking into account the symmetry, that there are two values of u with opposite signs, minimizing ground state energy, and indicating on its two-fold degeneration. Physically the third part of Hamiltonian describes electron-phonon interaction with total constant $\alpha = \alpha_1 + \alpha_2$, which a priori seems to be extending the range of applicability of SSH-model to essentially more strong values of electron-phonon interaction because electron-electron interactions are known (from quantum chemical studies of small polyenes) to be strong.

Operator $\hat{\mathcal{H}}(u)$ is invariant under spatial translations with period $2a$, where a is projection of spacing between two adjacent CH-groups in undimerized lattice on chain axis direction, and which is equal to 1.22 $\mathring{A}$. It means, that all various wave vectors $\vec{k}$ in $\vec{k}$-space will be in reduced zone with module of $\vec{k}$ in the range $-\frac{\pi}{2a} \le k \le \frac{\pi}{2a}$ [12]. It can be considered like to usual semiconductors to be consisting of two subzones - conduction (c) band and valence (v) band. Then it seems to be convenient to represent the operators $\{\hat{c}^+_{m,s}\}$, $\{\hat{c}_{m,s}\}$, $m = \overline{1,N}$, in the form

$$\{\hat{c}_{m,s}\} = \{\hat{c}^{(c)}_{m,s}\} + \{\hat{c}^{(v)}_{m,s}\}, \quad \{\hat{c}^+_{m,s}\} = \{\hat{c}^{+(c)}_{m,s}\} + \{\hat{c}^{+(v)}_{m,s}\}, \quad (10)$$

related to $\pi - c$- and $\pi - v$-band correspondingly, and to define $\vec{k}$-space operators

$$\{\hat{c}^{(c)}_{k,s}\} = \{\frac{i}{\sqrt{N}}\sum_m \sum_s (-1)^{m+1}\exp(-ikma)\hat{c}^{(c)}_{m,s}\}, \quad \{\hat{c}^{(v)}_{k,s}\} = \{\frac{1}{\sqrt{N}}\sum_m \sum_s \exp(-ikma)\hat{c}^{(v)}_{m,s}\}, \quad (11)$$

$m = \overline{1,N}$. The principle, like to MO LCAO is used in fact to build the operators $\{\hat{c}^{(c)}_{k,s}\}$ and $\{\hat{c}^{(v)}_{k,s}\}$, at that

the antibonding character of c-band orbitals is taken into account by means of factor $i(-1)^{m+1}$. Inverse to (11) transform is

$$\{\hat{c}^{(c)}_{m,s}\} = \{\frac{1}{\sqrt{N}}\sum_k \exp i[m(ka+\pi) - \frac{\pi}{2}]\hat{c}^{(c)}_{k,s}\},$$
$$\{\hat{c}^{(v)}_{m,s}\} = \{\frac{1}{\sqrt{N}}\sum_k \exp(ikma)\hat{c}^{(v)}_{k,s}\}, \quad (12)$$

$m = \overline{1,N}$.

The σ-operators $\{\hat{a}^{+}_{m,s}\}$ and $\{\hat{a}_{m,s}\}$, $m = \overline{1,N}$ can also be represented in the form like to (10) for π-operators and analogous to (11), (12) transforms can be defined. Then the expression for the operator $\hat{\mathcal{H}}_0(u)$ can be rewritten

$$\hat{\mathcal{H}}_0(u) = \hat{\mathcal{H}}^{\sigma,c}_0(u) + \hat{\mathcal{H}}^{\sigma,v}_0(u) = \sum_m\sum_s(\frac{\hat{P}^2_m}{2M^*} + Ku^2_m)\times$$
$$\frac{1}{N}\sum_k(\hat{a}^{+\sigma,c}_{k,s}\hat{a}^{\sigma,c}_{k,s} + \hat{a}^{+\sigma,v}_{k,s}\hat{a}^{\sigma,v}_{k,s}), \quad (13)$$

where $\hat{a}^{+\sigma,c}_{k,s}$, $\hat{a}^{\sigma,c}_{k,s}$ and $\hat{a}^{+\sigma,v}_{k,s}$, $\hat{a}^{\sigma,v}_{k,s}$ are σ-operators of creation and annihilation, related to σ-c-band and to σ-v-band correspondingly. The independence of $|u_m|$ on m, $m = \overline{1,N}$, means, that the expression $(\frac{\hat{P}^2_m}{2M^*} + Ku^2_m)$ is independent on m. Then we obtain

$$\hat{\mathcal{H}}_0(u) = \sum_k\sum_s(\frac{\hat{P}^2}{2M^*} + Ku^2)(\hat{n}^{\sigma,c}_{k,s} + \hat{n}^{\sigma,v}_{k,s}), \quad (14)$$

where $\hat{n}^{\sigma,c}_{k,s}$ and $\hat{n}^{\sigma,v}_{k,s}$ are operators of number of σ-quasiparticles in σ-c-band and σ-v-band correspondingly.

The expression for $\hat{\mathcal{H}}_{\pi,t}(u)$ in terms of $\{\hat{c}^{(c)}_{k,s}\}$ and $\{\hat{c}^{(v)}_{k,s}\}$ is coinciding with known corresponding expression in [11], [12] and it is

$$\hat{\mathcal{H}}_{\pi,t}(u) = \sum_k\sum_s 2t_0\cos ka(\hat{c}^{+(c)}_{k,s}\hat{c}^{(c)}_{k,s} - \hat{c}^{+(v)}_{k,s}\hat{c}^{(v)}_{k,s}) \quad (15)$$

The expression for the first part of operator $\hat{\mathcal{H}}_{\pi,u}(u)$ in terms of $\{\hat{c}^{(c)}_{k,s}\}$ and $\{\hat{c}^{(v)}_{k,s}\}$ is also coinciding in its form with known corresponding expression in [11], [12] and it is given by

$$\hat{\mathcal{H}}_{\pi,u,\alpha_1}(u) = \sum_k\sum_s 4\alpha_1 u\sin ka(\hat{c}^{+(v)}_{k,s}\hat{c}^{(c)}_{k,s} + \hat{c}^{+(c)}_{k,s}\hat{c}^{(v)}_{k,s}), \quad (16)$$

where subscript α_1 in Hamiltonian designation indicates on the taking into account the part of electron-phonon interaction, connected with resonance interaction (hopping) processes. The expression for the second part of operator $\hat{\mathcal{H}}_{\pi,u}(u)$, which describes the part of electron-phonon interaction, determined by interaction between quasiparticles in Fermi liquid state of π-subsystem in terms of $\{\hat{c}^{(c)}_{k,s}\}$ and $\{\hat{c}^{(v)}_{k,s}\}$ is the following

$$\hat{\mathcal{H}}_{\pi,u,\alpha_2}(u) = \sum_k\sum_{k'}\sum_s \alpha_2(k,k',s)\hat{c}^{+(c)}_{k',s}\hat{c}^{+(v)}_{k',s}\hat{c}^{(v)}_{k,s}\hat{c}^{(c)}_{k,s}. \quad (17)$$

Here the consideration is restricted by the taking into account the contribution of the term, corresponding to interaction between the quasiparticles in different bands, which seems to be the most essential. Physically the identification of linear on displacement u part of both resonance interaction (hopping) and the pairwise interaction of quasiparticles in π-subsystem between themselves with electron-phonon interaction is understandable, if to take into account, that by atomic CH group displacements the phonons are generated, which in its turn can by release of the place on, for instance, $(CH)_m$ group, to deliver the energy and impulse, which are necessary for transfer of the quasiparticle (electron) from adjacent $(m-1)$- or $(m+1)$-position in chain in the case of resonance interaction (hopping). For the case the pairwise interaction of quasiparticles, it means, that its linear on displacement u part is realized by means of phonon field, which transfers the energy and impulse from one quasiparticle to another (which can be not inevitable adjacent). Mathematically it can be proved in the following way. The processes of interaction in c (v) band can be considered to be independent on each other. It means, that transition probability from the $\langle k_{l,s}|$-state to $\langle k_{j,s}|$-state in c-band and from $\langle k'_{l,s}|$-state to $\langle k'_{j,s}|$-state in v-band, which is proportional to coefficient $\alpha_2(k,k',s)$, can be expressed in the form of product of real parts of corresponding matrix elements, that is in the form

$$\alpha_2(k,k',s) \sim Re\langle k_{l,s}|\hat{V}^{(c)}|k_{j,s}\rangle Re\langle k'_{l,s}|\hat{V}^{(v)}|k'_{j,s}\rangle =$$
$$\sum_{k_{ph}} Re\langle k_{l,s}|\hat{V}^{(c)}|k_{ph}\rangle\langle k_{ph}|k_{j,s}\rangle\times$$
$$\sum_{k_{ph}} Re\langle k'_{r,s}|\hat{V}^{(v)}|k_{ph}\rangle\langle k_{ph}|k'_{n,s}\rangle, \quad (18)$$

where $\hat{V}^{(v)} = V_{0(v)}\hat{e}$ ($\hat{e}$ is unit operator) is the first term in Taylor expansion of pairwise interaction of quasiparticles, for instance, with wave vectors k'_r, k'_n and spin projection s in v-band, that is, in ground state, $\hat{V}^{(c)} = V_{1(c)}u\hat{e}$ is the second term in Taylor expansion of pairwise interaction in excited state (in c-band), that is, it is product of configuration coordinate u and coordinate derivative at $u=0$ of operator of pairwise interaction of quasiparticles with wave vectors k_l, k_j and spin projection s in c-band, k_{ph} is phonon wave vector, and the summation is realized over all the phonon spectrum. At that, since the linear density of pairwise interaction is independent on k, which is the consequence of translation invariance of the chain, $V_{0(v)}$, $V_{1(c)}$ are constants. Therefore, the pairwise interaction is considered to be accompanying by process of phonon generation, when electronic quasiparticles are already in excited state, that is, in c-band

(retardation effect of phonon subsystem is taken into account). Then we have $\hat{V}^{(c)} = V_{0(c)}u\hat{e}$, $\hat{V}^{(v)} = V_{0(v)}\hat{e}$. A number of variants are possible along with process of phonon generation, corresponding to states of electronic quasiparticles in c-band above described. The result will mathematically be quite similar, if to change the energetic place of excitation, that is, if to interchange the role of c and v bands for given process. There seem to be possible the realization of both the stages (that is phonon generation and absorption) for electonic quasiparticles in single c or v band states and simultaneous realization both the stages in both the bands. Mathematical description will be for all possible variants similar and for distinctness we will consider only the first variant. For the simplicity we consider also the processes, in which the spin projection is keeping to be the same. It is evident also, that in z-direction the impulse distribution is quasicontinuous (since the chain has the macroscopic length $L = Na$). Consequently, standard way $\sum_{k_{ph}} \to \frac{L}{2\pi}\int_{k_{ph}}$ can be used. Further, phonon states can be described by wave functions $\langle k_{ph}| = v_0 exp(ik_{ph}z)$, where $z \in [0, L]$, $k_{ph} \in [-\frac{\pi}{2a}, \frac{\pi}{2a}]$, v_0 is constant. Therefore, from (18) we have the expression

$$\alpha_2(k,k',s) = b|v_{0v}|^2|v_{0c}|^2V_{0(c)}uV_{0(v)}|\phi_{0cs}|^2|\phi_{0vs}|^2\times \frac{N}{2\pi(q_l-q_j)(q_r-q_n)}Re\{\exp[i(k_lm_l-k_jm_j)a]\exp ika\}\times Re\{\exp[i(k'_rm_r-k'_nm_n)a]\exp ik'a\}, \tag{19}$$

where $|\phi_{0cs}|^2$, $|\phi_{0vs}|^2$ are squares of the modules of the wave functions $|k_{j,s}\rangle$ and $|k'_{j,s}\rangle$ respectively, $k = k_{ph}(q_l - q_j)$, $k' = k'_{ph}(q_r - q_n)$ $q_l, q_j, q_r, q_n \in N$ with additional conditions $(q_l - q_j)a \le L$, $(q_r - q_n)a \le L$, b - is aspect ratio, which in principle can be determined by comparison with experiment. Here the values $(q_l - q_j)$, $(q_r - q_n)$ determine the steps in pairwise interaction with phonon participation and they are considered to be fixed. We will consider the processes for which $k = k'$, consequently, $(q_r - q_n) = (q_l - q_j)$.

The relation (19) by $k_lm_l = k_jm_j$ and by $k_rm_r = k_nm_n$ transforms into

$$\alpha_2(k,k',s) = b|v_{0v}|^2|v_{0c}|^2V_{0(c)}uV_{0(v)}|\phi_{0cs}|^2|\phi_{0vs}|^2\times \frac{N}{2\pi[(q_l-q_j)]^2}\sin ka\sin k'a. \tag{20}$$

Let us designate

$$b|v_{0v}|^2|v_{0c}|^2V_{0(c)}V_{0(v)}|\phi_{0cs}|^2|\phi_{0vs}|^2\times \frac{N}{2\pi[(q_l-q_j)]^2} = 4\alpha_2(s) \tag{21}$$

Then, taking into account that spin projection s is fixed, the dependence on s can be omitted, consequently $\alpha_2(s) = \alpha_2$. So we have

$$\hat{\mathcal{H}}_{\pi,u,\alpha_2}(u) = \sum_k\sum_{k'}\sum_s 4\alpha_2 u\sin ka\sin k'a\hat{c}^{+(c)}_{k',s}\hat{c}^{+(v)}_{k',s}\hat{c}^{(v)}_{k,s}\hat{c}^{(c)}_{k,s}. \tag{22}$$

Something otherwise will be treated the participation of the phonons in linear on u part of pairwise interaction, if phonon generation is accompanying process of quasiparticle transition from v-band into c-band, that is when the retardation effect of phonon subsystem can be neglected. It is the case of strong electron-photon interaction, described in [23], [14], [22]. By strong electron-photon interation absorption process of photons is long time process. It is accompanying by quantum Rabi wave packet formation and space propagation, that is by formation of longlived coherent state of joint photon-electron system. In given case the expression for density of the electron-phonon coupling parameter $\alpha_2(k,k',s)$, which is related to the part of electron-phonon interaction, determined by interaction between quasiparticles in π-system Fermi liquid, is the following

$$\alpha_2(k,k',s) \sim Re\langle k_{l,s}|\hat{V}|k'_{j,s}\rangle = |v_{0v}|^2|v_{0c}|^2uV_1|\phi_{0s}|^2\times \frac{N^2}{[2\pi]^2}\int_{k_{ph}}\exp[i(k_{ph}qa - k_lm_la)]\times \{\int_{k'_{ph}}\exp[i(k'_{ph}-k_{ph})q'a]\times \exp[-i(k'_{ph}q'a - k'_jm_na)]dk'_{ph}\}dk_{ph}, \tag{23}$$

where $q = m_j - m_l$, $q' = m_r - m_n$ are integers, satisfying foregoing relations, subscrips in left part are omitted, since fixed step is considered. Then, taking into account, that in continuous limit by integration the modules k and k' of wave vectors $\vec{k}$ and $\vec{k'}$ are running all the k- and k'-values in k- and k'-spaces, we can designate $(k_{ph}qa - k_lm_la) = ka$, $(k'_{ph}q'a - k'_jm_ja) = k'a$ omitting the subscrips. In a result we obtain

$$\alpha_2(k,k',s) \sim Re\langle k_{l,s}|\hat{V}|k'_{j,s}\rangle = |v_{0v}|^2|v_{0c}|^2uV_1|\phi_{0s}|^2\times \frac{N^2}{[2\pi]^2}(\sin ka\sin k'a + \cos ka\cos k'a). \tag{24}$$

It was taken into account, that by v-band $\to$ c-band transition of quasiparticle, the impulse of emitted phonon by vibronic system with electronic quasiparticle in v-band is equal to the impulse of absorbed phonon by vibronic system with electronic quasiparticle in c-band.

The system of operators $\hat{c}^{+(c)}_{k',s}$, $\hat{c}^{+(v)}_{k',s}$, $\hat{c}^{(v)}_{k,s}$, $\hat{c}^{(c)}_{k,s}$ corresponds to noninteracting quasiparticles, and it is understandable, that in the case of interacting quasiparticles their linear combination has to be used

$$\begin{bmatrix}\hat{a}^{(v)}_{k,s}\\ \hat{a}^{(c)}_{k,s}\end{bmatrix} = \begin{bmatrix}\alpha_{k,s} & -\beta_{k,s}\\ \beta_{k,s} & \alpha_{k,s}\end{bmatrix}\begin{bmatrix}\hat{c}^{(v)}_{k,s}\\ \hat{c}^{(c)}_{k,s}\end{bmatrix}, \tag{25}$$

where matrix of transformation coefficients $||A||$ is

$$||A|| = \begin{bmatrix} \alpha_{k,s} & -\beta_{k,s} \\ \beta_{k,s} & \alpha_{k,s}, \end{bmatrix} \tag{26}$$

it is unimodulary matrix with determinant $det||A|| = \alpha_{k,s}^2 + \beta_{k,s}^2 = 1$. Since $det||A|| \neq 0$, it means, that inverse transformation exists and it is given by the matrix

$$||A||^{-1} = \begin{bmatrix} \alpha_{k,s} & \beta_{k,s} \\ -\beta_{k,s} & \alpha_{k,s} \end{bmatrix}. \tag{27}$$

Then we obtain for the Hamiltonian $\hat{\mathcal{H}}_{\pi,u,\alpha_1}(u)$, which corresponds to SSH one-electron treatment of electron-phonon coupling, the following relation

$$\begin{aligned}
&\hat{\mathcal{H}}_{\pi,u,\alpha_1}(u) = \\
&\sum_k \sum_s \Delta_k [\alpha_{k,s}^2 \hat{a}_{k,s}^{+(v)} \hat{a}_{k,s}^{(c)} - \alpha_{k,s}\beta_{k,s} \hat{a}_{k,s}^{+(v)} \hat{a}_{k,s}^{(v)} \\
&+ \beta_{k,s}\alpha_{k,s} \hat{a}_{k,s}^{+(c)} \hat{a}_{k,s}^{(c)} - \beta_{k,s}^2 \hat{a}_{k,s}^{+(c)} \hat{a}_{k,s}^{(v)} + \alpha_{k,s}^2 \hat{a}_{k,s}^{+(c)} \hat{a}_{k,s}^{(v)} \\
&+ \alpha_{k,s}\beta_{k,s} \hat{a}_{k,s}^{+(c)} \hat{a}_{k,s}^{(c)} - \beta_{k,s}\alpha_{k,s} \hat{a}_{k,s}^{+(v)} \hat{a}_{k,s}^{(v)} - \beta_{k,s}^2 \hat{a}_{k,s}^{+(v)} \hat{a}_{k,s}^{(c)}],
\end{aligned} \tag{28}$$

where $\Delta_k = 4\alpha_1 u \sin ka$. The diagonal part $\hat{\mathcal{H}}_{\pi,u,\alpha_1}^d(u)$ of operator $\hat{\mathcal{H}}_{\pi,u,\alpha_1}(u)$ is

$$\hat{\mathcal{H}}_{\pi,u,\alpha_1}^d(u) = \sum_k \sum_s 2\Delta_k \alpha_{k,s}\beta_{k,s} (\hat{n}_{k,s}^{(c)} - \hat{n}_{k,s}^{(v)}), \tag{29}$$

where $\hat{n}_{k,s}^{(c)}$ is density of operator of quasiparticles' number in c-band, $\hat{n}_{k,s}^{(v)}$ is density of operator of quasiparticles' number in v-band. The part of pairwise interaction, which is linear in displacement coordinate u and leads to participation of the phonons, is given by the Hamiltonian

$$\begin{aligned}
&\hat{\mathcal{H}}_{\pi,u,\alpha_2}(u) = \sum_k \sum_{k'} \sum_s 4\alpha_2 u \sin ka \sin k'a \times \\
&(\alpha_{k',s}^2 \hat{a}_{k',s}^{+(c)} \hat{a}_{k',s}^{(v)} - \beta_{k',s}^2 \hat{a}_{k',s}^{(v)} \hat{a}_{k',s}^{+(c)} \\
&+ \alpha_{k',s}\beta_{k',s} \hat{a}_{k',s}^{(c)} \hat{a}_{k',s}^{+(c)} - \beta_{k',s}\alpha_{k',s} \hat{a}_{k',s}^{(v)} \hat{a}_{k',s}^{+(v)}) \\
&\times (\alpha_{k,s}^2 \hat{a}_{k,s}^{+(c)} \hat{a}_{k,s}^{(v)} - \beta_{k,s}^2 \hat{a}_{k,s}^{+(v)} \hat{a}_{k,s}^{(c)} \\
&+ \alpha_{k,s}\beta_{k,s} \hat{a}_{k,s}^{+(c)} \hat{a}_{k,s}^{(c)} - \beta_{k,s}\alpha_{k,s} \hat{a}_{k,s}^{+(v)} \hat{a}_{k,s}^{(v)}).
\end{aligned} \tag{30}$$

The diagonal part $\hat{\mathcal{H}}_{\pi,u,\alpha_2}^d(u)$ of operator $\hat{\mathcal{H}}_{\pi,u,\alpha_2}(u)$ is

$$\begin{aligned}
&\hat{\mathcal{H}}_{\pi,u,\alpha_2}^d(u) = 4\alpha_2 u \sum_k \sum_{k'} \sum_s \alpha_{k'}\beta_{k'} (\hat{n}_{k',s}^{(v)} - \hat{n}_{k',s}^{(c)}) \\
&\times \alpha_{k,s}\beta_{k,s} (\hat{n}_{k,s}^{(v)} - \hat{n}_{k,s}^{(c)}) \sin k'a \sin ka
\end{aligned} \tag{31}$$

The Hamiltonian $\hat{\mathcal{H}}_{\pi,t}(u)$ in terms of operator variables $\hat{a}_{k,s}^{(c)}$ $\hat{a}_{k,s}^{(v)}$ is

$$\begin{aligned}
&\hat{\mathcal{H}}_{\pi,t}(u) = \sum_k \sum_s 2t_0 \cos ka [(\alpha_{k,s}^2 - \beta_{k,s}^2)(\hat{a}_{k,s}^{+(c)} \hat{a}_{k,s}^{(c)} - \\
&\hat{a}_{k,s}^{+(v)} \hat{a}_{k,s}^{(v)}) - 2\alpha_{k,s}\beta_{k,s} (\hat{a}_{k,s}^{+(v)} \hat{a}_{k,s}^{(c)} + \hat{a}_{k,s}^{+(c)} \hat{a}_{k,s}^{(v)})]
\end{aligned} \tag{32}$$

The diagonal part $\hat{\mathcal{H}}_{\pi,t}^d(u)$ of operator $\hat{\mathcal{H}}_{\pi,t}(u)$ is given by the relation

$$\hat{\mathcal{H}}_{\pi,t}^d(u) = \sum_k \sum_s \epsilon_k (\alpha_{k,s}^2 - \beta_{k,s}^2)(\hat{n}_{k,s}^{(c)} - \hat{n}_{k,s}^{(v)}), \tag{33}$$

where $\epsilon_k = 2t_0 \cos ka$.

The operator transformation for the σ-subsystem, analogous to (25) shows, that the Hamiltonian $\hat{\mathcal{H}}_0(u)$ is invariant under given transformation, that is, it can be represented in the form, given by (14).

To find the values of elements of matrices $||A||$ and $||A||^{-1}$, the Hamiltonian $\hat{\mathcal{H}}(u)$ has to be tested for conditional extremum on the variables α_k, β_k with condition $\alpha_{k,s}^2 + \beta_{k,s}^2 = 1$. The corresponding Lagrange function $\hat{\mathfrak{E}}^L(u)$ is

$$\begin{aligned}
&\hat{\mathfrak{E}}^L(u) = \sum_k \sum_s (\frac{\hat{P}^2}{2M^*} + Ku^2)(\hat{n}_{k,s}^{\sigma,c} + \hat{n}_{k,s}^{\sigma,v}) \\
&+ \sum_k \sum_s \epsilon_k (\alpha_{k,s}^2 - \beta_{k,s}^2)(\hat{n}_{k,s}^{(c)} - \hat{n}_{k,s}^{(v)}) \\
&+ \sum_k \sum_s 2\Delta_k \alpha_{k,s}\beta_{k,s} (\hat{n}_{k,s}^{(c)} - \hat{n}_{k,s}^{(v)}) \\
&+ 4\alpha_2 u \sum_k \sum_{k'} \sum_s \alpha_{k',s}\beta_{k',s} (\hat{n}_{k',s}^{(c)} - \hat{n}_{k',s}^{(v)}) \alpha_{k,s}\beta_{k,s} \\
&\times (\hat{n}_{k,s}^{(c)} - \hat{n}_{k,s}^{(v)}) \sin k'a \sin ka + \lambda_{k,s}(\alpha_{k,s}^2 - 1 + \beta_{k,s}^2)
\end{aligned} \tag{34}$$

Then, the necessary condition for extremum is determined by Lagrange equations

$$\begin{aligned}
&\frac{\partial \hat{\mathfrak{E}}^L(u)}{\partial \alpha_k} = 2\alpha_{k,s}\epsilon_k (\hat{n}_{k,s}^{(c)} - \hat{n}_{k,s}^{(v)}) + 2\Delta_k \beta_{k,s} (\hat{n}_{k,s}^{(c)} - \hat{n}_{k,s}^{(v)}) \\
&\times [1 + \frac{\alpha_2}{\alpha_1} \sum_{k'} \sum_s \alpha_{k',s}\beta_{k',s} \sin k'a (\hat{n}_{k',s}^{(c)} - \hat{n}_{k',s}^{(v)})] \\
&+ 2\lambda_{k,s}\alpha_{k,s} = 0,
\end{aligned} \tag{35}$$

$$\begin{aligned}
&\frac{\partial \hat{\mathfrak{E}}^L(u)}{\partial \beta_{k,s}} = 2\beta_{k,s}\epsilon_k (\hat{n}_{k,s}^{(v)} - \hat{n}_{k,s}^{(c)}) + 2\Delta_k \alpha_{k,s} (\hat{n}_{k,s}^{(c)} - \hat{n}_{k,s}^{(v)}) \\
&\times [1 + \frac{\alpha_2}{\alpha_1} \sum_{k'} \sum_s \alpha_{k',s}\beta_{k',s} \sin k'a (\hat{n}_{k',s}^{(c)} - \hat{n}_{k',s}^{(v)})] \\
&+ 2\lambda_{k,s}\beta_{k,s} = 0
\end{aligned} \tag{36}$$

and

$$\frac{\partial \hat{\mathfrak{E}}^L(u)}{\partial \lambda_{k,s}} = \alpha_{k,s}^2 - 1 + \beta_{k,s}^2 = 0. \tag{37}$$

Let us designate

$$[1 + \frac{\alpha_2}{\alpha_1} \sum_{k'} \sum_s \alpha_{k',s}\beta_{k',s} \sin k'a (\hat{n}_{k',s}^{(c)} - \hat{n}_{k',s}^{(v)})] = \hat{\mathcal{Q}}, \tag{38}$$

then, passing on to observables in the Lagrange equations (35) - (37), we obtain for $\beta^2_{k,s}$, $\alpha^2_{k,s}$ and for product $\alpha_{k,s}\beta_{k,s}$ the relations

$$\beta^2_{k,s} = \frac{1}{2}(1 \pm \frac{\epsilon_k}{\sqrt{\epsilon_k^2 + \mathcal{Q}^2\Delta_k^2}}), \tag{39}$$

$$\alpha^2_{k,s} = \frac{1}{2}(1 \mp \frac{\epsilon_k}{\sqrt{\epsilon_k^2 + \mathcal{Q}^2\Delta_k^2}}), \tag{40}$$

$$\alpha_{k,s}\beta_{k,s} = \frac{1}{2}\frac{\mathcal{Q}\Delta_k}{\sqrt{\epsilon_k^2 + \mathcal{Q}^2\Delta_k^2}}, \tag{41}$$

where $\mathcal{Q}$ is eigenvalue of operator $\hat{\mathcal{Q}}$. The equation for factor $\mathcal{Q}$ is

$$[1 + \frac{\alpha_2}{2\alpha_1}\sum_k\sum_s \frac{\mathcal{Q}\Delta_k \sin ka}{\sqrt{\epsilon_k^2 + \mathcal{Q}^2\Delta_k^2}}(n^{(c)}_{k,s} - n^{(v)}_{k,s})] = \mathcal{Q}, \tag{42}$$

where superscript ' is omitted and $n^{(c)}_{k,s}$ is eigenvalue of density operator of quasiparticles' number in c-band, $n^{(v)}_{k,s}$ is eigenvalue of density operator of quasiparticles' number in v-band. It is evident, that at $Q = 1$ in (39) - (41) we have the case of SSH-model. It corresponds to the case, if $\frac{\alpha_2}{\alpha_1}\sum_k\sum_s \frac{1}{2}\frac{\Delta_k}{\sqrt{\epsilon_k^2+\Delta_k^2}}\sin ka(n^{(c)}_{k,s} - n^{(v)}_{k,s})] \to 0$, which is realized, if $\alpha_2 \to 0$. Consequently, it seems to be interesting to consider the opposite case, when $|\frac{\alpha_2}{\alpha_1}\sum_k\sum_s \frac{1}{2}\frac{\Delta_k}{\sqrt{\epsilon_k^2+\Delta_k^2}}\sin ka(n^{(c)}_{k,s} - n^{(v)}_{k,s})]| \gg 1$. Passing on to continuum limit, in which $\sum_k\sum_s \to 2\frac{Na}{\pi}\int\limits_0^{\frac{\pi}{2a}}$, and assuming $n^{(v)}_{k,s} = 1$, $n^{(c)}_{k,s} = 0$, we have integral equation

$$\frac{2Nua\alpha_2}{\alpha_1\pi t_0}\int\limits_0^{\frac{\pi}{2a}} \frac{\sin^2 ka}{\sqrt{1 - \sin^2 ka[1 - (\frac{2u\mathcal{Q}}{t_0})^2]}}dk = 1. \tag{43}$$

In the case $|\frac{2u\mathcal{Q}}{t_0}| < 1$ the relation (43) can be rewritten in the form

$$K\left\{\sqrt{1 - \left(\frac{2\alpha_1 u\mathcal{Q}}{t_0}\right)^2}\right\} - E\left\{\sqrt{1 - \left(\frac{2\alpha_1 u\mathcal{Q}}{t_0}\right)^2}\right\} = \frac{\pi[t_0^2 - (2u\mathcal{Q})^2]}{2Nu\alpha_2}, \tag{44}$$

where $K\left\{\sqrt{1 - (\frac{2\alpha_1 u\mathcal{Q}}{t_0})^2}\right\}$ and $E\left\{\sqrt{1 - (\frac{2u\mathcal{Q}}{t_0})^2}\right\}$ are complete elliptic integrals of the first and the second kind, respectively. Expanding given integrals into the series and restricting by the terms of the second-order of smallness, we obtain

$$\mathcal{Q} \approx \frac{t_0}{6u}\sqrt{25 - 32\frac{t_0\alpha_1}{Nu\alpha_2}}. \tag{45}$$

It is evident, that the condition $(\frac{2u\mathcal{Q}}{t_0}) < 1$ is held true by $\frac{1}{3}\sqrt{25 - 32\frac{t_0\alpha_1}{Nu\alpha_2}} < 1$.

In the case $|\frac{2u\mathcal{Q}}{t_0}| > 1$ the relation (43) can be represented in the form

$$\int\limits_0^{\frac{\pi}{2}} \frac{\cos^2 y}{\sqrt{1 - \sin^2 y[1 - (\frac{t_0}{2u\mathcal{Q}})^2]}}dy = -\frac{\pi\mathcal{Q}\alpha_1}{\alpha_2 N}, \tag{46}$$

where $y = \frac{\pi}{2} - ka$. It is equivalent to the equation

$$\left(\frac{t_0}{2u\mathcal{Q}}\right)F\left\{\frac{\pi}{2}, \sqrt{1 - \left(\frac{t_0}{2u\mathcal{Q}}\right)^2}\right\} - E\left\{\frac{\pi}{2}, \sqrt{1 - \left(\frac{t_0}{2u\mathcal{Q}}\right)^2}\right\} = \frac{\pi\mathcal{Q}\alpha_1}{\alpha_2 N}\left[1 - \left(\frac{t_0}{2u\mathcal{Q}}\right)^2\right], \tag{47}$$

where $F\left\{\frac{\pi}{2}, \sqrt{1 - \left(\frac{t_0}{2u\mathcal{Q}}\right)^2}\right\}$ is the complete elliptic integral of the first kind. The approximation of elliptic integrals, like to the approximation, given by (45), leads to the relation

$$\mathcal{Q} \approx \frac{-3\alpha_2 N}{16}\left[1 \pm \sqrt{1 + \frac{80\alpha_1 t_0}{9Nu\alpha_2}}\right]. \tag{48}$$

In the case $\frac{2u\mathcal{Q}}{t_0} = 1$ the relation (43) is

$$\int\limits_0^{\frac{\pi}{2}} \cos^2 y dy = -\frac{\pi\alpha_1\mathcal{Q}}{\alpha_2 N}, \tag{49}$$

where $y = \frac{\pi}{2} - ka$. It is seen, that in given case the value of parameter Q is calculated exactly and it is

$$\mathcal{Q} = \frac{\alpha_2 N}{4\alpha_1} \tag{50}$$

The values of energy of π-quasiparticles in c-band $E^{(c)}_k(u)$ and in v-band $E^{(v)}_k(u)$ can be obtained in the following way

$$E^{(c)}_k(u) = \frac{\partial\mathfrak{E}^L(u)}{\partial n^{(c)}_{k,s}}, E^{(v)}_k(u) = \frac{\partial\mathfrak{E}^L(u)}{\partial n^{(v)}_{k,s}}, \tag{51}$$

where $\mathfrak{E}^L(u)$ is the energy of π-subsystem of t-PA chain, which is obtained from Lagrange function operator (34)

by passing on to observables. Therefore, we have

$$E_k^{(c)}(u) = \epsilon_k(\alpha_{k,s}^2 - \beta_{k,s}^2) + 2\Delta_k\alpha_{k,s}\beta_{k,s} + 8\alpha_2 u \sin ka \times \sum_{k'}\sum_s \alpha_{k',s}\beta_{k',s}(\hat{n}_{k',s}^{(c)} - \hat{n}_{k',s}^{(v)}) \sin k'a\alpha_{k,s}\beta_{k,s} = \epsilon_k(\alpha_{k,s}^2 - \beta_{k,s}^2) + 2\Delta_k\alpha_{k,s}\beta_{k,s}\mathcal{Q}) \quad (52)$$

and

$$E_k^{(v)}(u) = -\epsilon_k(\alpha_{k,s}^2 - \beta_{k,s}^2) - 2\Delta_k\alpha_{k,s}\beta_{k,s} - 8\frac{\alpha_2}{u}\sin ka \times \sum_{k'}\sum_s \alpha_{k',s}\beta_{k',s}(\hat{n}_{k',s}^{(c)} - \hat{n}_{k',s}^{(v)}) \sin k'a\alpha_{k,s}\beta_{k,s} = -\epsilon_k(\alpha_{k,s}^2 - \beta_{k,s}^2) - 2\Delta_k\alpha_{k,s}\beta_{k,s}\mathcal{Q}). \quad (53)$$

It is seen from (52) and (53), that $E_k^{(v)}(u) = -E_k^{(c)}(u)$. Taking into account the relations (39) - (41), we obtain

$$E_k^{(c)}(u) = \mp\frac{\epsilon_k^2}{\sqrt{\epsilon_k^2 + \mathcal{Q}^2\Delta_k^2}}) + \frac{\mathcal{Q}^2\Delta_k^2}{\sqrt{\epsilon_k^2 + \mathcal{Q}^2\Delta_k^2}}, \quad (54)$$

$$E_k^{(c)}(u) = \pm\frac{\epsilon_k^2}{\sqrt{\epsilon_k^2 + \mathcal{Q}^2\Delta_k^2}}) - \frac{\mathcal{Q}^2\Delta_k^2}{\sqrt{\epsilon_k^2 + \mathcal{Q}^2\Delta_k^2}}. \quad (55)$$

Therefore we have two values for the enegy of quasiparticles, indicating on the possibility of formation of the quasiparticles of two kinds. Upper sign in the first terms in (54), (55) correspond to the quasiparticles with the energy

$$E_k^{(c)}(u) = \frac{\mathcal{Q}^2\Delta_k^2 - \epsilon_k^2}{\sqrt{\epsilon_k^2 + \mathcal{Q}^2\Delta_k^2}}, \quad E_k^{(v)}(u) = \frac{\epsilon_k^2 - \mathcal{Q}^2\Delta_k^2}{\sqrt{\epsilon_k^2 + \mathcal{Q}^2\Delta_k^2}} \quad (56)$$

in c-band and v-band respectively. Lower sign in the first terms in (54), (55) correspond to the quasiparticles with the energy

$$E_k^{(c)}(u) = \sqrt{\epsilon_k^2 + \mathcal{Q}^2\Delta_k^2}, \quad E_k^{(v)}(u) = -\sqrt{\epsilon_k^2 + \mathcal{Q}^2\Delta_k^2} \quad (57)$$

in c-band and v-band respectively. The quasiparticles of the second kind at $\mathcal{Q} = 1$ are the same quasiparticles, that were obtained in [12].

We have used the only necessary condition for extremum of the functions $E(\alpha_{k,s}\beta_{k,s})$. It was shown in [22], that for the SSH-model the sufficient conditions for the minimum are substantial, they change the role of both solutions. Sufficient conditions for the minimum of the functions $E(\alpha_{k,s}\beta_{k,s})$ can be obtained by standard way, which was used in [22]. It consist in that, that the second differential of the energy being to be the function of three variables $\alpha_{k,s}$, $\beta_{k,s}$ and $\lambda_{k,s}$ has to be positively defined quadratic form. From the condition of positiveness of three principal minors of quadratic form coefficients we obtain the following three sufficient conditions for the energy minimum

a. The first condition The first condition is

$$\{\epsilon_k(1 - \frac{\epsilon_k}{\sqrt{\epsilon_k^2 + \mathcal{Q}^2\Delta_k^2}}) < \frac{(\mathcal{Q}\Delta)_k^2}{\sqrt{\epsilon_k^2 + \mathcal{Q}^2\Delta_k^2}}|(n_{k,s}^c - n_{k,s}^v) < 0\}, \quad \{\epsilon_k(1 - \frac{\epsilon_k}{\sqrt{\epsilon_k^2 + \mathcal{Q}^2\Delta_k^2}} > \frac{(\mathcal{Q}\Delta)_k^2}{\sqrt{\epsilon_k^2 + \mathcal{Q}^2\Delta_k^2}}|(n_{k,s}^c - n_{k,s}^v) > 0\} \quad (58)$$

for the solution which coincides with SSH-solution at $\mathcal{Q} = 1$ (SSH-like) solution and

$$\{\epsilon_k(1 + \frac{\epsilon_k}{\sqrt{\epsilon_k^2 + \mathcal{Q}^2\Delta_k^2}} < \frac{\mathcal{Q}\Delta_k^2}{\sqrt{\epsilon_k^2 + \mathcal{Q}^2\Delta_k^2}}|(n_{ks}^c - n_{ks}^v) < 0\}, \quad \{\epsilon_k(1 + \frac{\epsilon_k}{E_k} > \frac{(\mathcal{Q}\Delta)_k^2}{\sqrt{\epsilon_k^2 + \mathcal{Q}^2\Delta_k^2}}|(n_{ks}^c - n_{ks}^v) > 0\} \quad (59)$$

for the additional solution. It is seen, that the first condition is realizable for the quasiparticles of both the kinds, at that for both near equilibrium state ($n_{ks}^c - n_{ks}^v < 0$) and strongly nonequilibrium state ($n_{ks}^c - n_{ks}^v > 0$.

b. The second condition The second condition is the same for both the solutions and it is

$$(\frac{\epsilon_k^2}{\sqrt{\epsilon_k^2 + \mathcal{Q}^2\Delta_k^2}} - 2\frac{\mathcal{Q}\Delta_k^2}{\sqrt{\epsilon_k^2 + \mathcal{Q}^2\Delta_k^2}})^2 - \epsilon_k^2 + \frac{1}{4}(\mathcal{Q}\Delta)_k^2 > 0 \quad (60)$$

It is realizable for the quasiparticles of both the kinds.

c. The third condition For the SSH-like solution we have

$$(3\frac{(\mathcal{Q}\Delta)_k^2}{\sqrt{\epsilon_k^2 + \mathcal{Q}^2\Delta_k^2}} + 4\frac{\epsilon_k^2}{\sqrt{\epsilon_k^2 + \mathcal{Q}^2\Delta_k^2}})(n_{ks}^c - n_{ks}^v) > 0. \quad (61)$$

It means, that SSH-like solution is unapplicable for the description of standard processes, passing near equilibrium state by any parameters. The quasiparticles, described by SSH-like solution, can be created the only in strongly nonequilibrium state with inverse population of the levels in c- and v-bands. At the same time the solution, which corresponds to upper signs in (54), has to satisfy to the following condition

$$(3\frac{(\mathcal{Q}\Delta)_k^2}{\sqrt{\epsilon_k^2 + \mathcal{Q}^2\Delta_k^2}} - 4\frac{\epsilon_k^2}{\sqrt{\epsilon_k^2 + \mathcal{Q}^2\Delta_k^2}})(n_{ks}^c - n_{ks}^v) > 0, \quad (62)$$

which can be realized both in near equilibrium and in strongly nonequilibrium states of the π-subsystem of t-PA, which is considered to be quantum Fermi liquid.

d. Ground State of t-PA chain The continuum limit for the ground state energy of the t-PA chain with SSH-like quasiparticles will coincide with known solution [12],

if to replace $\Delta_k \mathcal{Q} \to \Delta_k$. Let us calculate the ground state energy $E_0^{[u]}(u)$ of the t-PA chain with quasiparticles' branch, which is stable near equilibrium. Taking into account, that in ground state $n_{k,s}^c = 0$, $n_{k,s}^v = 1$, in the continuum limit we have

$$E_0^{[u]}(u) = -\frac{2Na}{\pi} \int\limits_0^{\frac{\pi}{2a}} \frac{(\mathcal{Q}\Delta_k)^2 - \epsilon_k^2}{\sqrt{\mathcal{Q}\Delta_k)^2 + \epsilon_k^2}} dk + 2NKu^2, \quad (63)$$

then, calculating the integral and using the complete elliptic integral of the first kind $F(\frac{\pi}{2}, 1-z^2)$ and the complete elliptic integral of the second kind $E(\frac{\pi}{2}, 1-z^2)$ we obtain

$$E_0^{[u]}(u) = \frac{4Nt_0}{\pi}\{F(\frac{\pi}{2}, 1-z^2) + \frac{1+z^2}{1-z^2}[E(\frac{\pi}{2}, 1-z^2) - F(\frac{\pi}{2}, 1-z^2)]\} + 2NKu^2, \quad (64)$$

where $z^2 = \frac{2\mathcal{Q}\alpha_1 u}{t_0}$. Approximation of (64) at $z \ll 1$ gives

$$E_0^{[u]}(u) = N\{\frac{4t_0}{\pi} - \frac{6}{\pi} \ln \frac{2t_0}{\mathcal{Q}\alpha_1 u} \frac{4(\mathcal{Q}\alpha_1)^2 u^2}{t_0} + \frac{28(\mathcal{Q}\alpha_1)^2 u^2}{\pi t_0} + ...\} + 2NKu^2. \quad (65)$$

It is seen from (65), that the energy of quasiparticles, described by solution, which corresponds to upper signs in (54) has the form of Coleman-Weinberg potential with two minima at the values of dimerization coordinate u_0 and $-u_0$ like to the energy of quasiparticles, described by SSH-solution [12]. It is understandable, that subsequent consideration, including electrically neutral S=1/2 soliton and electrically charged spinless soliton formation, that is the appearence of the phenomenon of spin-charge separation, by Fermi liquid description of 1D systems will be coinciding in its mathematical form with starting intermediate SSH model.

Thus, the possibility to describe the physical properties of 1D systems in the frames of 1D quantum Fermi liquid including the mechanism of appearence of the most prominent feature of 1D systems - the phenomenon of spin-charge separation - is proved.

Therefore, all qualitative conclusions of the model proposed in [12] are holding in Fermi-liquid consideration of π-electronic subsystem of t-PA chain (instead Fermi-gas consideration) for the quasiparticles, corresponding to the second-branch-solution. It seems to be substantial, that Fermi-liquid treatment of electron-phonon interaction extends the applicability limits of SSH-model of 1D conjugated conductors allowing its use in the case of strong electron-phonon interaction. It is evident, that the mechanism of the phenomenon of spin-charge separation in 1D Fermi-liquid is soliton mechanism, being to be analogous to mechanism proposed by Jackiw and Rebbi. It means like to SSH-Fermi-gas model, that when an electron is added to an neutral polyacetylene chain, it can break up into two pieces, one of which carries the electron's charge and the other its spin. Given result bears a clear family relation with the phenomenon of spin-charge separation in the 1D electron gas theory of Luther and Emery [8], but it is quite different from Anderson spinon-holon mechanism. The results obtained make more exact and correct prevalent viewpoint, that spin-charge separation effect is indication on non-Fermi-liquid behavior of electronic systems and that it can be reasonably described in the frames of Luttinger liquid theory only. Given viewpoint is true the only for the systems, for which Anderson mechanism of spin-charge separation is realized, that is in fact for the systems with the function of the electronic energy in k-space, which characterises by the absence of extremum. It seem to be relatively rare physical systems.

The consideration proposed allows to extend the limits of the applicability of SSH model for description of the systems with both strong electron-phonon interaction and (or) strong electron-photon interaction.

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Quantum Fermi Liquid Decription of 1D Electronic Systems

Alla Dovlatova (a), Dmitri Yerchuck (b), Felix Borovik (b)
(a) - M.V.Lomonosov Moscow State University, Moscow, 119899,
(b) Heat-Mass Transfer Institute of National Academy of Sciences of RB, Brovka Str.15, Minsk, 220072
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The concept of quantum Fermi liquid for description of 1D electronic systems is recovered. The model of 1D quantum Fermi liquid is developed on the example of *trans*-polyacetylene and it is in given case the generalization of well-known Fermi gas Su-Schrieffer-Heeger (SSH) model of organic 1D conductors. It is shown, that spin-charge separation effect can be realized in 1D quantum Fermi liquids. It has topological soliton origin in distinction from well known spinon-holon spin-charge separation effect in Luttinger liquids and electronic systems like them. The model allows to extend the limits of the applicability of SSH-model to the systems with strong electron-phonon interaction and (or) strong electron-photon interaction. Practical significance of the model proposed consists in the clarification of the nature of charge and spin carriers and in the clarification of the origin of mechanisms of quasiparticles' interaction in the materials and objects of nanoelectronics, spintronics and the other nanotechnology branches.

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I. INTRODUCTION

There seems to be very essential for the tasks of nanoelectronics, spintronics and for the other branches of nanotechnology the knowledge of the nature of charge and spin carriers in the nano-devices. Especially seems to be significant the knowledge of mechanisms of carrier transport and interactions of charge and spin carriers both between themselves and with phonons and photons. In particular, great hopes are pinned in modern nanotechnology on carbon nanotubes (NTs), that is in using of carbon NTs for production of the main devices for nanoelectronics and related nanotechnology branches.

There is existing in the theory of 1D electronic systems, in particular in the theory of conducting NTs, the following concept, which was starting with the work of Tomonaga in 1950 [1] and with the work by Luttinger in 1963 [2], when it has become clear that the electron-electron interaction destroys the sharp Fermi surface and leads to a breakdown of the Landau Fermi liquid (LFL) theory. The resulting non-LFL state is commonly called Luttinger liquid (LL), or sometimes Tomonaga-Luttinger liquid. It describes the universal low-energy properties of 1D conductors. LL behavior is characterized by pronounced power-law suppression of the transport current and the density of states, and by effect of spin-charge separation. The nature of the spin and charge carriers in LL-liquid state is the folllowing. They are chargeless spin 1/2 quasiparticles - spinons and spinless quasiparticles with the charge ± e - holons. The universality of LL description means that the physical properties do not depend on details of the model, the interaction potential, and so on, but instead are only characterized by a few parameters - critical exponents. Quite remarkably, the LL concept is believed to hold for arbitrary statistical properties of the particles, that is, both for fermions and bosons. It provided a paradigm for non-Fermi liquid physics.

Let us remark, that it is argued in many works, that the single-wall carbon nanotubes (SWCNTs), considered to be 1D objects (it is not always correct, especially for standard NTs with diameter in several nanometers) can be described the only in the frame of LL concept. Moreover, SWCNTs are considered to be the best model system of the LL state demonstration. Given viewpoint has some grounds. Really, power-law behaviour was observed experimentally by measuring the tunneling conductance of SWNTs in dependence on temperature and voltage. Electron force microscopic measurements showed also the ballistic nature of transport in conducting SWNTs. At the same time spin-charge separation has not been observed so far. Consequently, we have to conclude, that the existing viewpoint seems to be insufficiently grounded.

It has to be also remarked, that both the models LL and LFL are the models of ideal quantum liquids (and even oversimplified), since they do not take into account the nonlinearity of the fermion spectrum on the one hand and electron-phonon interactions on the other hand. In fact both the models describe not strongly adequately the real physical processes. Actually, the changes in a charge state of arbitrary atom in 1D chain in the result of electron-electron interaction are always accompanied by the changes in phonon subsystem (and vice versa). It is consequence of generic coupling between operators of creation and annihilation in electron subsystem and in phonon field. Consequently, the models, which do not take into account the electron-phonon interaction, seem to be strongly oversimplified and restricted models. Further, the key argument for insertion of the notion "Luttinger liquid" is in fact also the simplification, determined by linearization of the generic spectrum of particles in neighborhood of Fermi points in k-space. Just given simplification has led to divergencies arising in the perturba-

tion theory in 1D-case, that is in LFL theory. However, it does not means that 1D Fermi liquid description is incorrect in general case. Consequently, the description of 1D systems, for instance NTs, the only in the frames of LL concept seems to be also oversimplification. We will show, that the concept of description of 1D correlated electronic systems in the frame of 1D Fermi liquid (FL) can be recovered, at that FL concept can be applied just to 1D carbon NTs, while 2D NTs and graphene can be in principle described in the low energy physics processes in the frame of LL model. It is the aim of the presented work.

We will consider the concept of 1D FL on the example of well known 1D system - *trans*-polyacetylene, that is, it will be in fact the generalization of well known Fermi gas (since it does not take into consideration electron-electron correlations immediately) Su-Schrieffer-Heeger (SSH) model, which, however, in distinction from LFL and LL models, takes into account the electron-phonon interaction. The subsequent generalization, for instance, for quasi-1D carbon zigzag shaped nanotubes (CZSNTS) can be easily obtained by using of hypercomplex number theory like to description of quantum optics effects, considered in [5], [6].

Spin-charge separation effect in 1D-systems is associated in the literature usually with Luttinger liquid (LL) behavior of electronic system of 1D conductors, which cannot be described in the frames of Landau Fermi liquid concept. The most known example of 1D-system with LL behaviour is doped Mott-Hubbard insulator in the metallic regime, for which the idea of spin-charge separation was originally introduced by Anderson in 1987 [7], [8], [9]. Spin-charge separation in Anderson approach means the existence of two independent elementary excitations, charge-neutral spinons and spinless holons, which carry spin $1/2$ and charge $+e$ respectively. Similar spin-charge separation effect may be mathematically realized in the so-called slave-particle representation [10] of the $t-J$ model.

Let us give some details concerning non-Fermi liquid physics paradigm. It has been developed from Tomonaga idea [1], that the low-energy degrees of freedom of a 1D Fermi gas are completely collective, that has allowed the development of the "bosonization" technique. The conceptual starting point for the bosonization of the Fermi surface is the Luttinger theorem [2], from here the name "Luttinger liquid" arose. However, how it was remarked in [11], even for a linear spectrum, the bosonic or fermionic languages may be used equally comfortably and both offer their particular benefits. The advantage of the former is the direct relation between the bosonic modes and the density response functions. On the other hand, the fermionic description connects to the well-known physics of the Fermi edge problem. Moreover, they have shown, that in order to calculate the dynamic response functions in the case of the nonlinearity of the fermion spectrum, it is convenient to translate the bosonic spin and charge modes into fermionic quasiparticles, spinons and holons. For a nonlinear spectrum, the fermionic basis is superior because it avoids divergencies arising in the bosonic perturbation theory [11].

At the same time the idea of spin-charge separation was explicitly treated for the first time already in 1974 by Luther and Emery [12] in the context of a continuum limit of the $1D$ electron gas theory. They have shown, that the Hamiltonian $\hat{H}_{1DEG}$ of the 1D electron gas can be represented in the form of

$$\hat{H}_{1DEG} = \hat{H}_c[\phi_c] + \hat{H}_s[\phi_s] + \hat{H}_{irr}[\phi_c, \phi_s], \qquad (1)$$

where $\hat{H}_c[\phi_c]$ and $\hat{H}_s[\phi_s]$ are, respectively, the Hamiltonians, which govern the dynamics of the spin and charge fields, ϕ_c and ϕ_s, respectively, and $\hat{H}_{irr}[\phi_c, \phi_s]$ consists of terms that can be neglected in the long wave-length limit.

The related model, which describes spin-charge separation is the model of the formation of solitons with fractional fermion number. General idea belongs to Jackiw and Rebbi [14]. They pay attention to the field theories, especially in one spatial dimension, which lead solitons' formation with fractional fermion number. However, the concrete realization of given idea in condensed matter physics belongs to Su, Schrieffer, and Heeger [3], [4]. The model, proposed by Su, Schrieffer, and Heeger (SSH-model) with spin-charge separation to be the basis phenomenon is the model of conjugated organic 1D-conductors.

Specifically, what Su, Schrieffer, and Heeger showed, is that when an electron is added to an neutral *trans*-polyacetylene (t-PA) chain, it can break up into two pieces, one of which carries the electron's charge and the other its spin. The real significance of the SSH-soliton model of t-PA is that it introduced a new paradigm into the field. Triuph of the SSH model is not occasional. The formulation of the model is very simple from mathematical viewpoint and the simplicity itself is the great advantage of the model. At the same time it demonstrates the deep physical insight of Su, Schrieffer and Heeger in the field, which was argued in [13], [5], [6]. Really, although the term, which takes into consideration the static electron-electron correlations is not presented in SSH Hamiltonian in explicit form, it, in fact, is represented in implicit form. Really, the static electron-electron interaction can be taken into account in the model by means of its renormalization into electron-phonon interaction with effective coupling parameter. It was undertaken in [16], [17]. It is very interesting, that the very similar theoretical result on the possibility to renormalize electron-phonon coupling into equivalent electron-electron static interactions was obtained independently many years after in 2006 in [21]. It was shown, that, the spin-1/2 Holstein model could be mapped onto the negative-U Hubbard model with an effective dynamical attraction $U_{eff}(\omega)$, dependence of which on the frequency ω is given by the relation $U_{eff}(\omega) = \lambda/(1-\omega^2/\omega_0^2)$, where λ is the electron-phonon coupling constant in energetic units, ω_0 is the

bare phonon frequency.

At the same time, although the model, used in [16] is the standard continuum model of a one-dimensional electron gas with short-range (that is screened) electron-electron repulsions, and a nearly half-filled band, however the very essential simplification has been done in given model. It consists in linearization of one electron spectrum about the Fermi surface, that seems to be oversiplification. Moreover, the idea of full renormalization of electron-electron interaction into electron-phonon interaction with effective coupling parameter seems to be correct the only partly (see next Section). It concerns also in principle the inverse task [21] above cited. The same extended Hubbard model, that in [16], was used in [17], however the exact bare phonon propagator at the renormalized electronic energy scale was obtained and used. It leads to different physical conclusions concerning the possibility of observation of charge density wave (CDW) - singlet superconductivity (SS) crossover in t-PA in comparison with work [16], where the approximate form of phonon propagator was used. Voit has concluded, that a CDW-SS crossover does not occur in the interacting SSH model in distinction from opposite conclusion in [16]. Let us remark, that in both the works above cited all the potential energy of electron-electron correlations is considered to be constant relatively the dimerization coordinate by renormalization procedure, it is also oversimplification, since in real physical processes the dimerization coordinate derivative of the potential energy of electron-electron correlations seems to be the most essential.

The merit of SSH-model, consisting in the choose of the only dimerization coordinate u_n of the n-th CH-group, $n = \overline{1, N}$, along chain molecular-symmetry axis x to be substantial for determination of main physical properties of the material and neglecting by the other five degrees of freedom, that is, the degrees of freedom, which are relevant to the bonds with the directions not coinciding with chain molecular-symmetry axis direction, was commented in [5] and in [6]. Given possibility is the consequence of general principle, which was proposed by Slater at the earliest stage of quantum physics era already in 1924 [18]. However, the most merit of SSH-model, which demonstrates a very deep insight of authors in the field, is without any comment up to now. In fact, the only given model in the condensed matter physics of dynamic electronic systems takes into consideration in explicit form the generic coupling between operators of creation and annihilation of two quantum fields - between the operators of the field corresponding to electron subsystem and the operators of the field of lattice deformation system, that is, phonon field. The simplest static analogue of taking into account the generic coupling between given two fields is quantum chemistry calculations of the structure of point centers in crystals. It is well known, that by the change of the charge state of any point center in crystal lattice, the atomic relaxation of neighbourhood host lattice atoms has to be taken into account. In dynamical case it corresponds to phonon absorption or generation. It seems to be evident, that in SSH-model the operators of phonon subsystem are represented through operators of electronic subsystem taking into account given coupling in explicit form. Let us remark, that usually given operators are considered independently on each other, which can lead to distortion of description of real physical processes.

It seems to be interesting, that there are fundamental qualitative differences by description of spin-charge separation effects in 1D systems between SSH mechanism and Anderson mechanism, which is applicable, in particular, to the models, in which correlated electronic systems are properly described by the Luttinger liquid theory. The main differerence consists in the role of phonon effects in spin-charge separation phenomenon. For instance, the role of phonon effects on spin-charge separation in one dimension by Anderson mechanism was studied in [21] through the calculation of one-electron spectral functions in terms of the cluster perturbation theory together with an optimized phonon approach. The 1D Holstein-Hubbard model has been used. It was found, that the retardation effect, which is the consequence of the finiteness of phonon frequency, suppresses the spin-charge separation and eventually makes it invisible in the spectral function. At the same time electron-phonon interaction plays the essential role for spin-charge separation presence in SSH-model.

Let us remark, that there is in existing variant of SSH-model an upper limit on the value of electron-phonon coupling constant. It is consequence of the treatment of electron-phonon coupling to be the linear term in expansion of the only hopping integral of tight-binding model about the undimerized state. Given restriction was discussed in [22] and the maximum for allowed value of electron-phonon coupling constant α was evaluated to be ≈ 1.27. We will show, that given restriction can be remitted in developed variant of SSH-model.

Su, Schrieffer, Heeger [3], [4] describe mathematically the chain of t-PA by considering it to be Fermi gas in the sense, that the electron-electron interaction is not taken into consideretion in explicit form, although electron-phonon interaction is taken into account. We see, therefore, that SSH-model takes, more strictly speaking, the intermediate place between Fermi gas and Fermi liquid quantum models. The main idea of our work is the development of SSH-model in the frames of completely 1D Fermi liquid description, that determines the aim - to show the possibility to describe the 1D electronic systems in the frames of quantum Fermi liquids, and to establish the possibility of the phenomenon of spin charge separation in 1D quantum Fermi liquids.

II. RESULTS AND DISCUSSION

We will start from Hamiltonian

$$\hat{\mathcal{H}}(u) = \hat{\mathcal{H}}_0(u) + \hat{\mathcal{H}}_{\pi,t}(u) + \hat{\mathcal{H}}_{\pi,u}(u). \qquad (2)$$

Like to works [3], [4] we will consider Born-Oppenheimer approximation. The first term in (2) is the following

$$\hat{\mathcal{H}}_0(u) = \sum_m \sum_s (\frac{\hat{P}_m^2}{2M^*}\hat{a}_{m,s}^+\hat{a}_{m,s} + Ku_m^2\hat{a}_{m,s}^+\hat{a}_{m,s}). \quad (3)$$

It represents itself the sum of operator of kinetic energy of CH-group motion (the first term in (3)) and the operator of the σ-bonding energy (the second term). Coefficient K in (3) is effective σ-bonds spring constant, M^* is total mass of CH-group, u_m is configuration coordinate for m-th CH-group, which corresponds to translation of m-th CH-group along the symmetry axis z of the chain, $m = \overline{1,N}$, N is number of CH-groups in the chain, $\hat{P}_m$ is operator of impulse, conjugated to configuration coordinate u_m, $m = \overline{1,N}$, $\hat{a}_{m,s}^+$, $\hat{a}_{m,s}$ are creation and annihilation operators of creation or annihilation of quasipartile with spin projection s on the m-th chain site in σ-subsystem of t-PA. The second term in (2) can be represented in the form of two components and it is

$$\begin{aligned}&\hat{\mathcal{H}}_{\pi,t}(u) = \hat{\mathcal{H}}_{\pi,t_0}(u) + \hat{\mathcal{H}}_{\pi,\alpha_1}(u) = \\ &\sum_m \sum_s [(t_0(\hat{c}_{m+1,s}^+\hat{c}_{m,s} + \hat{c}_{m,s}^+\hat{c}_{m+1,s})] + \\ &(-1)^m 2\alpha_1 u)(\hat{c}_{m+1,s}^+\hat{c}_{m,s} + \hat{c}_{m,s}^+\hat{c}_{m+1,s})],\end{aligned} \quad (4)$$

where $\hat{c}_{m,s}^+$, $\hat{c}_{m,s}$ are creation and annihilation operators of creation or annihilation of quasiparticle with spin projection s on the m-th chain site in π-subsystem of t-PA. It is the resonance interaction (hopping interaction in tight-binding model approximation) of quasiparticles in π-subsystem of t-PA electronic system, which is considered to be Fermi liquid, and in which the only constant and linear terms in Taylor series expansion of resonance integral about the dimerized state are taking into account.

Operator $\hat{\mathcal{H}}(u)$ is invariant under spatial translations with period $2a$, where a is projection of spacing between two adjacent CH-groups in undimerized lattice on chain axis direction, and which is equal to 1.22 $\mathring{A}$. It means, that all various wave vectors $\vec{k}$ in $\vec{k}$-space will be in reduced zone with module of $\vec{k}$ in the range $-\frac{\pi}{2a} \le k \le \frac{\pi}{2a}$ [4]. It can be considered like to usual semiconductors to be consisting of two subzones - conduction (c) band and valence (v) band. Then it seems to be convenient to represent the operators $\{\hat{c}_{m,s}^+\}$, $\{\hat{c}_{m,s}\}$, $m = \overline{1,N}$, in the form

$$\begin{aligned}\{\hat{c}_{m,s}\} &= \{\hat{c}_{m,s}^{(c)}\} + \{\hat{c}_{m,s}^{(v)}\}, \\ \{\hat{c}_{m,s}^+\} &= \{\hat{c}_{m,s}^{+(c)}\} + \{\hat{c}_{m,s}^{+(v)}\},\end{aligned} \quad (5)$$

related to $\pi - c$- and $\pi - v$-band correspondingly, and to define $\vec{k}$-space operators

$$\begin{aligned}\{\hat{c}_{k,s}^{(c)}\} &= \{\frac{i}{\sqrt{N}}\sum_m \sum_s (-1)^{m+1}\exp(-ikma)\hat{c}_{m,s}^{(c)}\}, \\ \{\hat{c}_{k,s}^{(v)}\} &= \{\frac{1}{\sqrt{N}}\sum_m \sum_s \exp(-ikma)\hat{c}_{m,s}^{(v)}\},\end{aligned} \quad (6)$$

$m = \overline{1,N}$. The principle, like to MO LCAO is used in fact to build the operators $\{\hat{c}_{k,s}^{(c)}\}$ and $\{\hat{c}_{k,s}^{(v)}\}$, at that the antibonding character of c-band orbitals is taken into account by means of factor $i(-1)^{m+1}$. Inverse to (6) transform is

$$\begin{aligned}\{\hat{c}_{m,s}^{(c)}\} &= \{\frac{1}{\sqrt{N}}\sum_k \exp i[m(ka+\pi) - \frac{\pi}{2}]\hat{c}_{k,s}^{(c)}\}, \\ \{\hat{c}_{m,s}^{(v)}\} &= \{\frac{1}{\sqrt{N}}\sum_k \exp(ikma)\hat{c}_{k,s}^{(v)}\},\end{aligned} \quad (7)$$

$m = \overline{1,N}$.

The σ-operators $\{\hat{a}_{m,s}^+\}$ and $\{\hat{a}_{m,s}\}$, $m = \overline{1,N}$ can also be represented in the form like to (5) for π-operators and analogous to (6), transforms can be defined. Then the expression for the operator $\hat{\mathcal{H}}_0(u)$ can be rewritten

$$\begin{aligned}&\hat{\mathcal{H}}_0(u) = \hat{\mathcal{H}}_0^{\sigma,c}(u) + \hat{\mathcal{H}}_0^{\sigma,v}(u) = \sum_m \sum_s (\frac{\hat{P}_m^2}{2M^*} + Ku_m^2)\times \\ &\frac{1}{N}\sum_k (\hat{a}_{k,s}^{+\sigma,c}\hat{a}_{k,s}^{\sigma,c} + \hat{a}_{k,s}^{+\sigma,v}\hat{a}_{k,s}^{\sigma,v}),\end{aligned} \quad (8)$$

where $\hat{a}_{k,s}^{+\sigma,c}$, $\hat{a}_{k,s}^{\sigma,c}$ and $\hat{a}_{k,s}^{+\sigma,v}$, $\hat{a}_{k,s}^{\sigma,v}$ are σ-operators of creation and annihilation, related to σ-c-band and to σ-v-band correspondingly. The independence of $|u_m|$ on m, $m = \overline{1,N}$, means, that the expression $(\frac{\hat{P}_m^2}{2M^*} + Ku_m^2)$ is independent on m. Then we obtain

$$\hat{\mathcal{H}}_0(u) = \sum_k \sum_s (\frac{\hat{P}^2}{2M^*} + Ku^2)(\hat{n}_{k,s}^{\sigma,c} + \hat{n}_{k,s}^{\sigma,v}), \quad (9)$$

where $\hat{n}_{k,s}^{\sigma,c}$ and $\hat{n}_{k,s}^{\sigma,v}$ are operators of number of σ-quasiparticles in σ-c-band and σ-v-band correspondingly.

The expression for $\hat{\mathcal{H}}_{\pi,t_0}(u)$ in terms of $\{\hat{c}_{k,s}^{(c)}\}$ and $\{\hat{c}_{k,s}^{(v)}\}$ is coinciding with known corresponding expression in [3], [4] and it is

$$\hat{\mathcal{H}}_{\pi,t_0}(u) = \sum_k \sum_s 2t_0\cos ka(\hat{c}_{k,s}^{+(c)}\hat{c}_{k,s}^{(c)} - \hat{c}_{k,s}^{+(v)}\hat{c}_{k,s}^{(v)}) \quad (10)$$

The expression for the second part of operator $\hat{\mathcal{H}}_{\pi,t}(u)$ in terms of $\{\hat{c}_{k,s}^{(c)}\}$ and $\{\hat{c}_{k,s}^{(v)}\}$ is also coinciding in its form with known corresponding expression in [3], [4] and it is given by

$$\hat{\mathcal{H}}_{\pi,\alpha_1}(u) = \sum_k \sum_s 4\alpha_1 u\sin ka(\hat{c}_{k,s}^{+(v)}\hat{c}_{k,s}^{(c)} + \hat{c}_{k,s}^{+(c)}\hat{c}_{k,s}^{(v)}), \quad (11)$$

where subscript α_1 in Hamiltonian designation indicates on the taking into account the part of electron-phonon interaction, connected with resonance interaction (hopping) processes.

The σ-operators $\{\hat{a}_{m,s}^+\}$ and $\{\hat{a}_{m,s}\}$, $m = \overline{1,N}$ can also be represented in the form like to (5) for π-operators and

analogous to (6), (7) transforms can be defined. Then the expression for the operator $\hat{\mathcal{H}}_0(u)$ can be rewritten

$$\hat{\mathcal{H}}_0(u) = \hat{\mathcal{H}}_0^{\sigma,c}(u) + \hat{\mathcal{H}}_0^{\sigma,v}(u) = \sum_m \sum_s (\frac{\hat{P}_m^2}{2M^*} + Ku_m^2)\times \frac{1}{N}\sum_k (\hat{a}_{k,s}^{+\sigma,c}\hat{a}_{k,s}^{\sigma,c} + \hat{a}_{k,s}^{+\sigma,v}\hat{a}_{k,s}^{\sigma,v}), \tag{12}$$

where $\hat{a}_{k,s}^{+\sigma,c}$, $\hat{a}_{k,s}^{\sigma,c}$ and $\hat{a}_{k,s}^{+\sigma,v}$, $\hat{a}_{k,s}^{\sigma,v}$ are σ-operators of creation and annihilation, related to σ-c-band and to σ-v-band correspondingly. The independence of $|u_m|$ on m, $m = \overline{1,N}$, means, that the expression $(\frac{\hat{P}_m^2}{2M^*} + Ku_m^2)$ is independent on m. Then we obtain

$$\hat{\mathcal{H}}_0(u) = \sum_k \sum_s (\frac{\hat{P}^2}{2M^*} + Ku^2)(\hat{n}_{k,s}^{\sigma,c} + \hat{n}_{k,s}^{\sigma,v}), \tag{13}$$

where $\hat{n}_{k,s}^{\sigma,c}$ and $\hat{n}_{k,s}^{\sigma,v}$ are operators of number of σ-quasiparticles in σ-c-band and σ-v-band correspondingly.

The constant term in Taylor series expansion of potential energy of electron-electron interaction about the dimerization coordinate and the term proportional to dimerization coordinate derivative of potential energy of electron-electron interaction will be taken into account in given consideration, since they seems to be represent the most essential. It will be shown, that both the terms make the contribution in the expression for the $\hat{\mathcal{H}}_{\pi,u}(u)$, which describes the part of electron-phonon interaction, determined by interaction between quasiparticles in Fermi liquid state of π-subsystem in terms of $\{\hat{c}_{k,s}^{(c)}\}$ and $\{\hat{c}_{k,s}^{(v)}\}$. It can be represented in the form

$$\hat{\mathcal{H}}_{\pi,u}(u) = \sum_k \sum_{k'} \sum_s \alpha_2(k,k',s)\hat{c}_{k',s}^{+(c)}\hat{c}_{k',s}^{+(v)}\hat{c}_{k,s}^{(v)}\hat{c}_{k,s}^{(c)}. \tag{14}$$

(The constant static term, which is determined by interaction on different atomic sites in a chain corresponding to the constant terms in Taylor series expansion of potential energy of electron-electron interaction about the dimerization coordinate was omitted in given consideration, in order to establish the role of phonon assisted part).

Physically the identification of linear on displacement u part of both resonance interaction (hopping) and the pairwise interaction of quasiparticles in π-subsystem between themselves with electron-phonon interaction is understandable, if to take into account, that by atomic CH group displacements the phonons are generated, which in its turn can by release of the place on, for instance, $(CH)_m$ group, to deliver the energy and impulse, which are necessary for transfer of the quasiparticle (electron) from adjacent $(m-1)$- or $(m+1)$-position in chain in the case of resonance interaction (hopping). For the case the pairwise interaction of quasiparticles, it means, that its linear on displacement u part is realized by means of phonon field, which transfers the energy and impulse from one quasiparticle to another (which can be not inevitable adjacent). Mathematically it can be proved in the following way. The processes of interaction in c (v) band can be considered to be independent on each other. It means, that transition probability from the $\langle k_{l,s}|$-state to $\langle k_{j,s}|$-state in c-band and from $\langle k'_{l,s}|$-state to $\langle k'_{j,s}|$-state in v-band, which is proportional to coefficient $\alpha_2(k,k',s)$, can be expressed in the form of product of real parts of corresponding matrix elements, that is in the form

$$\alpha_2(k,k',s) \sim Re\langle k_{l,s}|\hat{V}^{(c)}|k_{j,s}\rangle Re\langle k'_{l,s}|\hat{V}^{(v)}|k'_{j,s}\rangle = \sum_{k_{ph}} Re\langle k_{l,s}|\hat{V}^{(c)}|k_{ph}\rangle\langle k_{ph}|k_{j,s}\rangle\times \sum_{k_{ph}} Re\langle k'_{r,s}|\hat{V}^{(v)}|k_{ph}\rangle\langle k_{ph}|k'_{n,s}\rangle, \tag{15}$$

where $\hat{V}^{(v)} = V_{0(v)}\hat{e}$ ($\hat{e}$ is unit operator) is the first term in Taylor expansion of pairwise interaction of quasiparticles, for instance, with wave vectors k'_r, k'_n and spin projection s in v-band, that is, in ground state, $\hat{V}^{(c)} = V_{1(c)}u\hat{e}$ is the second term in Taylor expansion of pairwise interaction in excited state (in c-band), that is, it is product of configuration coordinate u and coordinate derivative at $u = 0$ of operator of pairwise interaction of quasiparticles with wave vectors k_l, k_j and spin projection s in c-band, k_{ph} is phonon wave vector, and the summation is realized over all the phonon spectrum. At that, since the linear density of pairwise interaction is independent on k, which is the consequence of translation invariance of the chain, $V_{0(v)}$, $V_{1(c)}$ are constants. Therefore, the pairwise interaction is considered to be accompanying by process of phonon generation, when electronic quasiparticles are already in excited state, that is, in c-band (retardation effect of phonon subsystem is taken into account). Then we have $\hat{V}^{(c)} = V_{0(c)}u\hat{e}$, $\hat{V}^{(v)} = V_{0(v)}\hat{e}$. A number of variants are possible along with process of phonon generation, corresponding to states of electronic quasiparticles in c-band above described. The result will mathematically be quite similar, if to change the energetic place of excitation, that is, if to interchange the role of c and v bands for given process. There seem to be possible the realization of both the stages (that is phonon generation and absorption) for electonic quasiparticles in single c or v band states and simultaneous realization both the stages in both the bands. Mathematical description will be for all possible variants similar and for distinctness we will consider only the first variant. For the simplicity we consider also the processes, in which the spin projection is keeping to be the same. It is evident also, that in z-direction the impulse distribution is quasicontinuous (since the chain has the macroscopic length $L = Na$). Consequently, standard way $\sum_{k_{ph}} \to \frac{L}{2\pi}\int_{k_{ph}}$ can be used. Further, phonon states can be described by wave functions $\langle k_{ph}| = v_0 exp(ik_{ph}z)$, where $z \in [0,L]$, $k_{ph} \in [-\frac{\pi}{2a}, \frac{\pi}{2a}]$, v_0 is constant. Therefore, from (15) we

have the expression

$$\alpha_2(k,k',s) = b|v_{0v}|^2|v_{0c}|^2V_{0(c)}uV_{0(v)}|\phi_{0cs}|^2|\phi_{0vs}|^2\times \frac{N}{2\pi(q_l-q_j)(q_r-q_n)}Re\{\exp[i(k_lm_l-k_jm_j)a]\exp ika\}\times Re\{\exp[i(k'_rm_r-k'_nm_n)a]\exp ik'a\}, \tag{16}$$

where $|\phi_{0cs}|^2$, $|\phi_{0vs}|^2$ are squares of the modules of the wave functions $|k_{j,s}\rangle$ and $|k'_{j,s}\rangle$ respectively, $k = k_{ph}(q_l - q_j)$, $k' = k'_{ph}(q_r - q_n)$ $q_l, q_j, q_r, q_n \in N$ with additional conditions $(q_l - q_j)a \le L$, $(q_r - q_n)a \le L$, b - is aspect ratio, which in principle can be determined by comparison with experiment. Here the values $(q_l - q_j)$, $(q_r - q_n)$ determine the steps in pairwise interaction with phonon participation and they are considered to be fixed. We will consider the processes for which $k = k'$, consequently, $(q_r - q_n) = (q_l - q_j)$.

The relation (16) by $k_lm_l = k_jm_j$ and by $k_rm_r = k_nm_n$ transforms into

$$\alpha_2(k,k',s) = b|v_{0v}|^2|v_{0c}|^2V_{0(c)}uV_{0(v)}|\phi_{0cs}|^2|\phi_{0vs}|^2\times \frac{N}{2\pi[(q_l-q_j)]^2}\sin ka\sin k'a. \tag{17}$$

Let us designate

$$b|v_{0v}|^2|v_{0c}|^2V_{0(c)}V_{0(v)}|\phi_{0cs}|^2|\phi_{0vs}|^2\times \frac{N}{2\pi[(q_l-q_j)]^2} = 4\alpha_2(s) \tag{18}$$

Then, taking into account that spin projection s is fixed, the dependence on s can be omitted, consequently $\alpha_2(s) = \alpha_2$. So we have

$$\hat{\mathcal{H}}_{\pi,u}(u) = \sum_k\sum_{k'}\sum_s 4\alpha_2u\sin ka\sin k'a\hat{c}^{+(c)}_{k',s}\hat{c}^{+(v)}_{k',s}\hat{c}^{(v)}_{k,s}\hat{c}^{(c)}_{k,s}. \tag{19}$$

Something otherwise will be treated the participation of the phonons in linear on u part of pairwise interaction, if phonon generation is accompanying process of quasiparticle transition from v-band into c-band, that is when the retardation effect of phonon subsystem can be neglected. It is the case of strong electron-photon interaction, described in [23], [5], [6]. By strong electron-photon interation absorption process of photons is long time process. It is accompanying by quantum Rabi wave packet formation and space propagation, that is by formation of longlived coherent state of joint photon-electron system. In given case the expression for density of the electron-phonon coupling parameter $\alpha_2(k,k',s)$, which is related to the part of electron-phonon interaction, determined by interaction between quasiparticles in π-system Fermi liquid, is the following

$$\alpha_2(k,k',s) \sim Re\langle k_{l,s}|\hat{V}|k'_{j,s}\rangle = |v_{0v}|^2|v_{0c}|^2uV_1|\phi_{0s}|^2\times \frac{N^2}{[2\pi]^2}\int_{k_{ph}}\exp[i(k_{ph}qa-k_lm_la)]\times \{\int_{k'_{ph}}\exp[i(k'_{ph}-k_{ph})q'a]\times \exp[-i(k'_{ph}q'a-k'_jm_na)]dk'_{ph}\}dk_{ph}, \tag{20}$$

where $q = m_j - m_l$, $q' = m_r - m_n$ are integers, satisfying foregoing relations, subscrips in left part are omitted, since fixed step is considered. Then, taking into account, that in continuous limit by integration the modules k and k' of wave vectors $\vec{k}$ and $\vec{k'}$ are running all the k- and k'-values in k- and k'-spaces, we can designate $(k_{ph}qa - k_lm_la) = ka$, $(k'_{ph}q'a - k'_jm_ja) = k'a$ omitting the subscrips. In a result we obtain

$$\alpha_2(k,k',s) \sim Re\langle k_{l,s}|\hat{V}|k'_{j,s}\rangle = |v_{0v}|^2|v_{0c}|^2uV_1|\phi_{0s}|^2\times \frac{N^2}{[2\pi]^2}(\sin ka\sin k'a+\cos ka\cos k'a). \tag{21}$$

It was taken into account, that by v-band $\to$ c-band transition of quasiparticle, the impulse of emitted phonon by vibronic system with electronic quasiparticle in v-band is equal to the impulse of absorbed phonon by vibronic system with electronic quasiparticle in c-band.

The system of operators $\hat{c}^{+(c)}_{k',s}$, $\hat{c}^{+(v)}_{k',s}$, $\hat{c}^{(v)}_{k,s}$, $\hat{c}^{(c)}_{k,s}$ corresponds to noninteracting quasiparticles, and it is understandable, that in the case of interacting quasiparticles their linear combination has to be used

$$\begin{bmatrix}\hat{a}^{(v)}_{k,s}\\ \hat{a}^{(c)}_{k,s}\end{bmatrix} = \begin{bmatrix}\alpha_{k,s} & -\beta_{k,s}\\ \beta_{k,s} & \alpha_{k,s}\end{bmatrix}\begin{bmatrix}\hat{c}^{(v)}_{k,s}\\ \hat{c}^{(c)}_{k,s}\end{bmatrix}, \tag{22}$$

where matrix of transformation coefficients $||A||$ is

$$||A|| = \begin{bmatrix}\alpha_{k,s} & -\beta_{k,s}\\ \beta_{k,s} & \alpha_{k,s},\end{bmatrix} \tag{23}$$

it is unimodulary matrix with determinant $det||A|| = \alpha^2_{k,s} + \beta^2_{k,s} = 1$. Since $det||A|| \neq 0$, it means, that inverse transformation exists and it is given by the matrix

$$||A||^{-1} = \begin{bmatrix}\alpha_{k,s} & \beta_{k,s}\\ -\beta_{k,s} & \alpha_{k,s}\end{bmatrix}. \tag{24}$$

Then we obtain for the Hamiltonian $\hat{\mathcal{H}}_{\pi,\alpha_1}(u)$, which corresponds to SSH one-electron treatment of electron-phonon coupling, the following relation

$$\hat{\mathcal{H}}_{\pi,\alpha_1}(u) = \sum_k\sum_s\Delta_k[\alpha^2_{k,s}\hat{a}^{+(v)}_{k,s}\hat{a}^{(c)}_{k,s} - \alpha_{k,s}\beta_{k,s}\hat{a}^{+(v)}_{k,s}\hat{a}^{(v)}_{k,s} + \beta_{k,s}\alpha_{k,s}\hat{a}^{+(c)}_{k,s}\hat{a}^{(c)}_{k,s} - \beta^2_{k,s}\hat{a}^{+(c)}_{k,s}\hat{a}^{(v)}_{k,s} + \alpha^2_{k,s}\hat{a}^{+(c)}_{k,s}\hat{a}^{(v)}_{k,s} + \alpha_{k,s}\beta_{k,s}\hat{a}^{+(c)}_{k,s}\hat{a}^{(c)}_{k,s} - \beta_{k,s}\alpha_{k,s}\hat{a}^{+(v)}_{k,s}\hat{a}^{(v)}_{k,s} - \beta^2_{k,s}\hat{a}^{+(v)}_{k,s}\hat{a}^{(c)}_{k,s}], \tag{25}$$

where $\Delta_k = 4\alpha_1 u \sin ka$.

The diagonal part $\hat{\mathcal{H}}^d_{\pi,\alpha_1}(u)$ of operator $\hat{\mathcal{H}}_{\pi,\alpha_1}(u)$ is

$$\hat{\mathcal{H}}^d_{\pi,\alpha_1}(u) = \sum_k \sum_s 2\Delta_k \alpha_{k,s}\beta_{k,s}(\hat{n}^{(c)}_{k,s} - \hat{n}^{(v)}_{k,s}), \tag{26}$$

where $\hat{n}^{(c)}_{k,s}$ is density of operator of quasiparticles' number in c-band, $\hat{n}^{(v)}_{k,s}$ is density of operator of quasiparticles' number in v-band.

The part of pairwise interaction, which is linear in displacement coordinate u and leads to participation of the phonons, is given by the Hamiltonian

$$\begin{aligned}
&\hat{\mathcal{H}}_{\pi,u}(u) = \sum_k \sum_{k'} \sum_s 4\alpha_2 u \sin ka \sin k'a \times \\
&(\alpha^2_{k',s}\hat{a}^{+(c)}_{k',s}\hat{a}^{(v)}_{k',s} - \beta^2_{k',s}\hat{a}^{(v)}_{k',s}\hat{a}^{+(c)}_{k',s} \\
&+ \alpha_{k',s}\beta_{k',s}\hat{a}^{(c)}_{k',s}\hat{a}^{+(c)}_{k',s} - \beta_{k',s}\alpha_{k',s}\hat{a}^{(v)}_{k',s}\hat{a}^{+(v)}_{k',s}) \\
&\times (\alpha^2_{k,s}\hat{a}^{+(c)}_{k,s}\hat{a}^{(v)}_{k,s} - \beta^2_{k,s}\hat{a}^{+(v)}_{k,s}\hat{a}^{(c)}_{k,s} \\
&+ \alpha_{k,s}\beta_{k,s}\hat{a}^{+(c)}_{k,s}\hat{a}^{(c)}_{k,s} - \beta_{k,s}\alpha_{k,s}\hat{a}^{+(v)}_{k,s}\hat{a}^{(v)}_{k,s}).
\end{aligned} \tag{27}$$

The diagonal part $\hat{\mathcal{H}}^d_{\pi,u}(u)$ of operator $\hat{\mathcal{H}}_{\pi,u}(u)$ is

$$\begin{aligned}
&\hat{\mathcal{H}}^d_{\pi,u}(u) = 4\alpha_2 u \sum_k \sum_{k'} \sum_s \alpha_{k'}\beta_{k'}(\hat{n}^{(v)}_{k',s} - \hat{n}^{(c)}_{k',s}) \\
&\times \alpha_{k,s}\beta_{k,s}(\hat{n}^{(v)}_{k,s} - \hat{n}^{(c)}_{k,s}) \sin k'a \sin ka
\end{aligned} \tag{28}$$

The Hamiltonian $\hat{\mathcal{H}}_{\pi,t_0}(u)$ in terms of operator variables $\hat{a}^{(c)}_{k,s}$ $\hat{a}^{(v)}_{k,s}$ is

$$\begin{aligned}
&\hat{\mathcal{H}}_{\pi,t_0}(u) = \sum_k \sum_s 2t_0 \cos ka[(\alpha^2_{k,s} - \beta^2_{k,s})(\hat{a}^{+(c)}_{k,s}\hat{a}^{(c)}_{k,s} - \\
&\hat{a}^{+(v)}_{k,s}\hat{a}^{(v)}_{k,s}) - 2\alpha_{k,s}\beta_{k,s}(\hat{a}^{+(v)}_{k,s}\hat{a}^{(c)}_{k,s} + \hat{a}^{+(c)}_{k,s}\hat{a}^{(v)}_{k,s})]
\end{aligned} \tag{29}$$

The diagonal part $\hat{\mathcal{H}}^d_{\pi,t_0}(u)$ of operator $\hat{\mathcal{H}}_{\pi,t_0}(u)$ is given by the relation

$$\hat{\mathcal{H}}^d_{\pi,t_0}(u) = \sum_k \sum_s \epsilon_k(\alpha^2_{k,s} - \beta^2_{k,s})(\hat{n}^{(c)}_{k,s} - \hat{n}^{(v)}_{k,s}), \tag{30}$$

where $\epsilon_k = 2t_0 \cos ka$.

The operator transformation for the σ-subsystem, analogous to (22) shows, that the Hamiltonian $\hat{\mathcal{H}}_0(u)$ is invariant under given transformation, that is, it can be represented in the form, given by (9).

To find the values of elements of matrices $||A||$ and $||A||^{-1}$, the Hamiltonian $\hat{\mathcal{H}}(u)$ has to be tested for conditional extremum on the variables α_k, β_k with condition $\alpha^2_{k,s} + \beta^2_{k,s} = 1$. The corresponding Lagrange function $\hat{\mathfrak{E}}^L(u)$ is

$$\begin{aligned}
&\hat{\mathfrak{E}}^L(u) = \sum_k \sum_s (\frac{\hat{P}^2}{2M^*} + Ku^2)(\hat{n}^{\sigma,c}_{k,s} + \hat{n}^{\sigma,v}_{k,s}) \\
&+ \sum_k \sum_s \epsilon_k(\alpha^2_{k,s} - \beta^2_{k,s})(\hat{n}^{(c)}_{k,s} - \hat{n}^{(v)}_{k,s}) \\
&+ \sum_k \sum_s 2\Delta_k \alpha_{k,s}\beta_{k,s}(\hat{n}^{(c)}_{k,s} - \hat{n}^{(v)}_{k,s}) \\
&+ 4\alpha_2 u \sum_k \sum_{k'} \sum_s \alpha_{k',s}\beta_{k',s}(\hat{n}^{(c)}_{k',s} - \hat{n}^{(v)}_{k',s})\alpha_{k,s}\beta_{k,s} \\
&\times (\hat{n}^{(c)}_{k,s} - \hat{n}^{(v)}_{k,s}) \sin k'a \sin ka + \lambda_{k,s}(\alpha^2_{k,s} - 1 + \beta^2_{k,s})
\end{aligned} \tag{31}$$

Then, the necessary condition for extremum is determined by Lagrange equations

$$\begin{aligned}
&\frac{\partial \hat{\mathfrak{E}}^L(u)}{\partial \alpha_k} = 2\alpha_{k,s}\epsilon_k(\hat{n}^{(c)}_{k,s} - \hat{n}^{(v)}_{k,s}) + 2\Delta_k\beta_{k,s}(\hat{n}^{(c)}_{k,s} - \hat{n}^{(v)}_{k,s}) \\
&\times [1 + \frac{\alpha_2}{\alpha_1}\sum_{k'}\sum_s \alpha_{k',s}\beta_{k',s} \sin k'a(\hat{n}^{(c)}_{k',s} - \hat{n}^{(v)}_{k',s})] \\
&+ 2\lambda_{k,s}\alpha_{k,s} = 0,
\end{aligned} \tag{32}$$

$$\begin{aligned}
&\frac{\partial \hat{\mathfrak{E}}^L(u)}{\partial \beta_{k,s}} = 2\beta_{k,s}\epsilon_k(\hat{n}^{(v)}_{k,s} - \hat{n}^{(c)}_{k,s}) + 2\Delta_k\alpha_{k,s}(\hat{n}^{(c)}_{k,s} - \hat{n}^{(v)}_{k,s}) \\
&\times [1 + \frac{\alpha_2}{\alpha_1}\sum_{k'}\sum_s \alpha_{k',s}\beta_{k',s} \sin k'a(\hat{n}^{(c)}_{k',s} - \hat{n}^{(v)}_{k',s})] \\
&+ 2\lambda_{k,s}\beta_{k,s} = 0
\end{aligned} \tag{33}$$

and

$$\frac{\partial \hat{\mathfrak{E}}^L(u)}{\partial \lambda_{k,s}} = \alpha^2_{k,s} - 1 + \beta^2_{k,s} = 0. \tag{34}$$

Let us designate

$$[1 + \frac{\alpha_2}{\alpha_1}\sum_{k'}\sum_s \alpha_{k',s}\beta_{k',s} \sin k'a(\hat{n}^{(c)}_{k',s} - \hat{n}^{(v)}_{k',s})] = \hat{\mathcal{Q}}, \tag{35}$$

then, passing on to observables in the Lagrange equations (32) - (34), we obtain for $\beta^2_{k,s}$, $\alpha^2_{k,s}$ and for product $\alpha_{k,s}\beta_{k,s}$ the relations

$$\beta^2_{k,s} = \frac{1}{2}(1 \pm \frac{\epsilon_k}{\sqrt{\epsilon^2_k + \mathcal{Q}^2\Delta^2_k}}), \tag{36}$$

$$\alpha^2_{k,s} = \frac{1}{2}(1 \mp \frac{\epsilon_k}{\sqrt{\epsilon^2_k + \mathcal{Q}^2\Delta^2_k}}), \tag{37}$$

$$\alpha_{k,s}\beta_{k,s} = \frac{1}{2}\frac{\mathcal{Q}\Delta_k}{\sqrt{\epsilon^2_k + \mathcal{Q}^2\Delta^2_k}}, \tag{38}$$

where $\mathcal{Q}$ is eigenvalue of operator $\hat{\mathcal{Q}}$. The equation for factor $\mathcal{Q}$ is

$$[1+\frac{\alpha_2}{2\alpha_1}\sum_k\sum_s\frac{\mathcal{Q}\Delta_k\sin ka}{\sqrt{\epsilon_k^2+\mathcal{Q}^2\Delta_k^2}}(n_{k,s}^{(c)}-n_{k,s}^{(v)})]=\mathcal{Q}, \quad (39)$$

where superscript ' is omitted and $n_{k,s}^{(c)}$ is eigenvalue of density operator of quasiparticles' number in c-band, $n_{k,s}^{(v)}$ is eigenvalue of density operator of quasiparticles' number in v-band. It is evident, that at $Q=1$ in (36) - (38) we have the case of SSH-model. It corresponds to the case, if $\frac{\alpha_2}{\alpha_1}\sum_k\sum_s\frac{1}{2}\frac{\Delta_k}{\sqrt{\epsilon_k^2+\Delta_k^2}}\sin ka(n_{k,s}^{(c)}-n_{k,s}^{(v)})]\to 0$, which is realized, if $\alpha_2\to 0$. Consequently, it seems to be interesting to consider the opposite case, when $|\frac{\alpha_2}{\alpha_1}\sum_k\sum_s\frac{1}{2}\frac{\Delta_k}{\sqrt{\epsilon_k^2+\Delta_k^2}}\sin ka(n_{k,s}^{(c)}-n_{k,s}^{(v)})]|\gg 1$. Passing on to continuum limit, in which $\sum_k\sum_s\to 2\frac{Na}{\pi}\int\limits_0^{\frac{\pi}{2a}}$, and assuming $n_{k,s}^{(v)}=1$, $n_{k,s}^{(c)}=0$, we have integral equation

$$\frac{2Nua\alpha_2}{\alpha_1\pi t_0}\int\limits_0^{\frac{\pi}{2a}}\frac{\sin^2 ka}{\sqrt{1-\sin^2 ka[1-(\frac{2u\mathcal{Q}}{t_0})^2]}}dk=1. \quad (40)$$

In the case $|\frac{2u\mathcal{Q}}{t_0}|<1$ the relation (40) can be rewritten in the form

$$K\left\{\sqrt{1-\left(\frac{2\alpha_1 u\mathcal{Q}}{t_0}\right)^2}\right\}-E\left\{\sqrt{1-\left(\frac{2\alpha_1 u\mathcal{Q}}{t_0}\right)^2}\right\}=\frac{\pi[t_0^2-(2u\mathcal{Q})^2]}{2Nu\alpha_2}, \quad (41)$$

where $K\left\{\sqrt{1-(\frac{2\alpha_1 u\mathcal{Q}}{t_0})^2}\right\}$ and $E\left\{\sqrt{1-(\frac{2u\mathcal{Q}}{t_0})^2}\right\}$ are complete elliptic integrals of the first and the second kind, respectively. Expanding given integrals into the series and restricting by the terms of the second-order of smallness, we obtain

$$\mathcal{Q}\approx\frac{t_0}{6u}\sqrt{25-32\frac{t_0\alpha_1}{Nu\alpha_2}}. \quad (42)$$

It is evident, that the condition $(\frac{2u\mathcal{Q}}{t_0})<1$ is held true by $\frac{1}{3}\sqrt{25-32\frac{t_0\alpha_1}{Nu\alpha_2}}<1$.

In the case $|\frac{2u\mathcal{Q}}{t_0}|>1$ the relation (40) can be represented in the form

$$\int\limits_0^{\frac{\pi}{2}}\frac{\cos^2 y}{\sqrt{1-\sin^2 y[1-(\frac{t_0}{2u\mathcal{Q}})^2]}}dy=-\frac{\pi\mathcal{Q}\alpha_1}{\alpha_2 N}, \quad (43)$$

where $y=\frac{\pi}{2}-ka$. It is equivalent to the equation

$$\left(\frac{t_0}{2u\mathcal{Q}}\right)F\left\{\frac{\pi}{2},\sqrt{1-\left(\frac{t_0}{2u\mathcal{Q}}\right)^2}\right\}-E\left\{\frac{\pi}{2},\sqrt{1-\left(\frac{t_0}{2u\mathcal{Q}}\right)^2}\right\}=\frac{\pi\mathcal{Q}\alpha_1}{\alpha_2 N}\left[1-\left(\frac{t_0}{2u\mathcal{Q}}\right)^2\right], \quad (44)$$

where $F\left\{\frac{\pi}{2},\sqrt{1-\left(\frac{t_0}{2u\mathcal{Q}}\right)^2}\right\}$ is the complete elliptic integral of the first kind. The approximation of elliptic integrals, like to the approximation, given by (42), leads to the relation

$$\mathcal{Q}\approx\frac{-3\alpha_2 N}{16}\left[1\pm\sqrt{1+\frac{80\alpha_1 t_0}{9Nu\alpha_2}}\right]. \quad (45)$$

In the case $\frac{2u\mathcal{Q}}{t_0}=1$ the relation (40) is

$$\int\limits_0^{\frac{\pi}{2}}\cos^2 ydy=-\frac{\pi\alpha_1\mathcal{Q}}{\alpha_2 N}, \quad (46)$$

where $y=\frac{\pi}{2}-ka$. It is seen, that in given case the value of parameter Q is calculated exactly and it is

$$\mathcal{Q}=\frac{\alpha_2 N}{4\alpha_1} \quad (47)$$

The values of energy of π-quasiparticles in c-band $E_k^{(c)}(u)$ and in v-band $E_k^{(v)}(u)$ can be obtained in the following way

$$E_k^{(c)}(u)=\frac{\partial\mathfrak{E}^L(u)}{\partial n_{k,s}^{(c)}}, E_k^{(v)}(u)=\frac{\partial\mathfrak{E}^L(u)}{\partial n_{k,s}^{(v)}}, \quad (48)$$

where $\mathfrak{E}^L(u)$ is the energy of π-subsystem of t-PA chain, which is obtained from Lagrange function operator (31) by passing on to observables. Therefore, we have

$$\begin{aligned}E_k^{(c)}(u)&=\epsilon_k(\alpha_{k,s}^2-\beta_{k,s}^2)+2\Delta_k\alpha_{k,s}\beta_{k,s}+8\alpha_2 u\sin ka\\&\times\sum_{k'}\sum_s\alpha_{k',s}\beta_{k',s}(\hat{n}_{k',s}^{(c)}-\hat{n}_{k',s}^{(v)})\sin k'a\alpha_{k,s}\beta_{k,s}\\&=\epsilon_k(\alpha_{k,s}^2-\beta_{k,s}^2)+2\Delta_k\alpha_{k,s}\beta_{k,s}\mathcal{Q})\end{aligned} \quad (49)$$

and

$$\begin{aligned}E_k^{(v)}(u)&=-\epsilon_k(\alpha_{k,s}^2-\beta_{k,s}^2)-2\Delta_k\alpha_{k,s}\beta_{k,s}-8\frac{\alpha_2}{u}\sin ka\\&\times\sum_{k'}\sum_s\alpha_{k',s}\beta_{k',s}(\hat{n}_{k',s}^{(c)}-\hat{n}_{k',s}^{(v)})\sin k'a\alpha_{k,s}\beta_{k,s}\\&=-\epsilon_k(\alpha_{k,s}^2-\beta_{k,s}^2)-2\Delta_k\alpha_{k,s}\beta_{k,s}\mathcal{Q}).\end{aligned} \quad (50)$$

It is seen from (49) and (50), that $E_k^{(v)}(u) = -E_k^{(c)}(u)$. Taking into account the relations (36) - (38), we obtain

$$E_k^{(c)}(u) = \mp\frac{\epsilon_k^2}{\sqrt{\epsilon_k^2+\mathcal{Q}^2\Delta_k^2}}) + \frac{\mathcal{Q}^2\Delta_k^2}{\sqrt{\epsilon_k^2+\mathcal{Q}^2\Delta_k^2}}, \quad (51)$$

$$E_k^{(c)}(u) = \pm\frac{\epsilon_k^2}{\sqrt{\epsilon_k^2+\mathcal{Q}^2\Delta_k^2}}) - \frac{\mathcal{Q}^2\Delta_k^2}{\sqrt{\epsilon_k^2+\mathcal{Q}^2\Delta_k^2}}. \quad (52)$$

Therefore we have two values for the enegy of quasiparticles, indicating on the possibility of formation of the quasiparticles of two kinds. Upper sign in the first terms in (51), (52) correspond to the quasiparticles with the energy

$$\begin{aligned} E_k^{(c)}(u) &= \frac{\mathcal{Q}^2\Delta_k^2-\epsilon_k^2}{\sqrt{\epsilon_k^2+\mathcal{Q}^2\Delta_k^2}}, \\ E_k^{(v)}(u) &= \frac{\epsilon_k^2-\mathcal{Q}^2\Delta_k^2}{\sqrt{\epsilon_k^2+\mathcal{Q}^2\Delta_k^2}} \end{aligned} \quad (53)$$

in c-band and v-band respectively. Lower sign in the first terms in (51), (52) correspond to the quasiparticles with the energy

$$\begin{aligned} E_k^{(c)}(u) &= \sqrt{\epsilon_k^2+\mathcal{Q}^2\Delta_k^2}, \\ E_k^{(v)}(u) &= -\sqrt{\epsilon_k^2+\mathcal{Q}^2\Delta_k^2} \end{aligned} \quad (54)$$

in c-band and v-band respectively. The quasiparticles of the second kind at $\mathcal{Q} = 1$ are the same quasiparticles, that were obtained in [4].

We have used the only necessary condition for extremum of the functions $E(\alpha_{k,s}\beta_{k,s})$. It was shown in [6], that for the SSH-model the sufficient conditions for the minimum are substantial, they change the role of both solutions. Sufficient conditions for the minimum of the functions $E(\alpha_{k,s}\beta_{k,s})$ can be obtained by standard way, which was used in [6]. It consist in that, that the second differential of the energy being to be the function of three variables $\alpha_{k,s}$, $\beta_{k,s}$ and $\lambda_{k,s}$ has to be positively defined quadratic form. From the condition of positiveness of three principal minors of quadratic form coefficients we obtain the following three sufficient conditions for the energy minimum

a. The first condition The first condition is

$$\begin{aligned} &\{\epsilon_k(1-\frac{\epsilon_k}{\sqrt{\epsilon_k^2+\mathcal{Q}^2\Delta_k^2}}) < \frac{(\mathcal{Q}\Delta)_k^2}{\sqrt{\epsilon_k^2+\mathcal{Q}^2\Delta_k^2}}|(n_{k,s}^c - n_{k,s}^v) < 0\}, \\ &\{\epsilon_k(1-\frac{\epsilon_k}{\sqrt{\epsilon_k^2+\mathcal{Q}^2\Delta_k^2}} > \frac{(\mathcal{Q}\Delta)_k^2}{\sqrt{\epsilon_k^2+\mathcal{Q}^2\Delta_k^2}}|(n_{k,s}^c - n_{k,s}^v) > 0\} \end{aligned} \quad (55)$$

for the solution which coincides with SSH-solution at $\mathcal{Q} = 1$ (SSH-like) solution and

$$\begin{aligned} &\{\epsilon_k(1+\frac{\epsilon_k}{\sqrt{\epsilon_k^2+\mathcal{Q}^2\Delta_k^2}} < \frac{\mathcal{Q}\Delta_k^2}{\sqrt{\epsilon_k^2+\mathcal{Q}^2\Delta_k^2}}|(n_{ks}^c - n_{ks}^v) < 0\}, \\ &\{\epsilon_k(1+\frac{\epsilon_k}{E_k} > \frac{(\mathcal{Q}\Delta)_k^2}{\sqrt{\epsilon_k^2+\mathcal{Q}^2\Delta_k^2}}|(n_{ks}^c - n_{ks}^v) > 0\} \end{aligned} \quad (56)$$

for the additional solution. It is seen, that the first condition is realizable for the quasiparticles of both the kinds, at that for both near equilibrium state ($n_{ks}^c - n_{ks}^v < 0$) and strongly nonequilibrium state ($n_{ks}^c - n_{ks}^v > 0$.

b. The second condition The second condition is the same for both the solutions and it is

$$(\frac{\epsilon_k^2}{\sqrt{\epsilon_k^2+\mathcal{Q}^2\Delta_k^2}} - 2\frac{\mathcal{Q}\Delta_k^2}{\sqrt{\epsilon_k^2+\mathcal{Q}^2\Delta_k^2}})^2 - \epsilon_k^2 + \frac{1}{4}(\mathcal{Q}\Delta)_k^2 > 0 \quad (57)$$

It is realizable for the quasiparticles of both the kinds.

c. The third condition For the SSH-like solution we have

$$(3\frac{(\mathcal{Q}\Delta)_k^2}{\sqrt{\epsilon_k^2+\mathcal{Q}^2\Delta_k^2}} + 4\frac{\epsilon_k^2}{\sqrt{\epsilon_k^2+\mathcal{Q}^2\Delta_k^2}})(n_{ks}^c - n_{ks}^v) > 0. \quad (58)$$

It means, that SSH-like solution is unapplicable for the description of standard processes, passing near equilibrium state by any parameters. The quasiparticles, described by SSH-like solution, can be created the only in strongly nonequilibrium state with inverse population of the levels in c- and v-bands. At the same time the solution, which corresponds to upper signs in (51), has to satisfy to the following condition

$$(3\frac{(\mathcal{Q}\Delta)_k^2}{\sqrt{\epsilon_k^2+\mathcal{Q}^2\Delta_k^2}} - 4\frac{\epsilon_k^2}{\sqrt{\epsilon_k^2+\mathcal{Q}^2\Delta_k^2}})(n_{ks}^c - n_{ks}^v) > 0, \quad (59)$$

which can be realized both in near equilibrium and in strongly nonequilibrium states of the π-subsystem of t-PA, which is considered to be quantum Fermi liquid.

d. Ground State of t-PA chain The continuum limit for the ground state energy of the t-PA chain with SSH-like quasiparticles will coincide with known solution [4], if to replace $\Delta_k\mathcal{Q} \to \Delta_k$. Let us calculate the ground state energy $E_0^{[u]}(u)$ of the t-PA chain with quasiparticles' branch, which is stable near equilibrium. Taking into account, that in ground state $n_{k,s}^c = 0$, $n_{k,s}^v = 1$, in the continuum limit we have

$$E_0^{[u]}(u) = -\frac{2Na}{\pi}\int_0^{\frac{\pi}{2a}} \frac{(\mathcal{Q}\Delta_k)^2-\epsilon_k^2}{\sqrt{(\mathcal{Q}\Delta_k)^2+\epsilon_k^2}}dk + 2NKu^2, \quad (60)$$

then, calculating the integral and using the complete elliptic integral of the first kind $F(\frac{\pi}{2}, 1-z^2)$ and the complete elliptic integral of the second kind $E(\frac{\pi}{2}, 1-z^2)$ we

obtain

$$E_0^{[u]}(u) = \frac{4Nt_0}{\pi}\{F(\frac{\pi}{2}, 1-z^2)+ \\ \frac{1+z^2}{1-z^2}[E(\frac{\pi}{2}, 1-z^2) - F(\frac{\pi}{2}, 1-z^2)]\} + 2NKu^2, \quad (61)$$

where $z^2 = \frac{2\mathcal{Q}\alpha_1 u}{t_0}$. Approximation of (61) at $z \ll 1$ gives

$$E_0^{[u]}(u) = N\{\frac{4t_0}{\pi} - \frac{6}{\pi}\ln\frac{2t_0}{\mathcal{Q}\alpha_1 u}\frac{4(\mathcal{Q}\alpha_1)^2u^2}{t_0} + \\ \frac{28(\mathcal{Q}\alpha_1)^2u^2}{\pi t_0} + ...\} + 2NKu^2. \quad (62)$$

It is seen from (62), that the energy of quasiparticles, described by solution, which corresponds to upper signs in (51) has the form of Coleman-Weinberg potential with two minima at the values of dimerization coordinate u_0 and $-u_0$ like to the energy of quasiparticles, described by SSH-solution [4]. It is understandable, that subsequent consideration, including electrically neutral S=1/2 soliton and electrically charged spinless soliton formation, that is the appearence of the phenomenon of spin-charge separation, by Fermi liquid description of 1D systems will be coinciding in its mathematical form with starting intermediate SSH model.

Thus, the possibility to describe the physical properties of 1D systems in the frames of 1D quantum Fermi liquid including the mechanism of appearence of the most prominent feature of 1D systems - the phenomenon of spin-charge separation - is proved.

Therefore, all qualitative conclusions of the model proposed in [4] are holding in Fermi-liquid consideration of π-electronic subsystem of t-PA chain (instead Fermi-gas consideration) for the quasiparticles, corresponding to the second-branch-solution. It seems to be substantial, that Fermi-liquid treatment of electron-phonon interaction extends the applicability limits of SSH-model of 1D conjugated conductors allowing its use in the case of strong electron-phonon interaction. It is evident, that the mechanism of the phenomenon of spin-charge separation in 1D Fermi-liquid is soliton mechanism, being to be analogous to mechanism proposed by Jackiw and Rebbi. It means like to SSH-Fermi-gas model, that when an electron is added to an neutral polyacetylene chain, it can break up into two pieces, one of which carries the electron's charge and the other its spin. Given result bears a clear family relation with the phenomenon of spin-charge separation in the 1D electron gas theory of Luther and Emery [12], but it is quite different from Anderson spinon-holon mechanism. The results obtained make more exact and correct prevalent viewpoint, that spin-charge separation effect is indication on non-Fermi-liquid behavior of electronic systems and that it can be reasonably described in the frames of Luttinger liquid theory only. Given viewpoint is true the only for the systems, for which Anderson mechanism of spin-charge separation is realized, that is in fact for the systems with the function of the electronic energy in k-space, which characterises by the absence of extremum. It seem to be relatively rare physical systems.

The consideration proposed allows to extend the limits of the applicability of SSH model for description of the systems with both strong electron-phonon interaction and (or) strong electron-photon interaction.

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